ANALYSIS AND OPTIMIZATION OF THE MULTIEFFECT-MULTISTAGE FLASH DISTILLATION AND REVERSE OSMOSIS DESALINATION PROCESSES

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KOH-DON KIANG

B. S., National Taiwan University, 1963

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements of the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

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Approved by:

Liang-tseng Fan
Major Professor

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PART ONE

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CHAPTER 1

INTRODUCTION

The present study is directed to the System analysis and optimization of a multieffect multistage (MEMS) flash distillation process. The MEMS flash distillation process is a rather recent development in the flash distillation technology and offers the most promise in the foreseeable future for producing large quantities of potable water economically from seawater (1).

A better understanding of the MEMS system is obtained by following the developments of the process. Regular distillation is a familiar water purification process. Flash distillation was introduced because of better control of scale formation (2). In the flash distillation process, heated saline water is released into a closed vessel which is maintained at a lower pressure than the vapor pressure of the solution. Since the vapor simply flashes off the warm liquid, the resulting precipitates form in the liquid and not on the heat transfer surface (3).

The brine concentration in a flashing chamber is nearly uniform due to the vigorous mixing resulting from the flashing and is, therefore, equal to that of the discharge stream. In a single stage operation, the feed brine with low concentration is mixed with flashing brine with high concentration. This causes a large amount of free energy loss due to the irreversible mixing of two solutions with considerable concentration difference. However, the concentration difference between stages in a multistage operation is considerably reduced. Therefore, the thermodynamic efficiency for this operation is greatly improved. This multistage operation is the so-called "single-effect multistage (SEMS)" flash distillation process (4).

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A recirculated SEMS is a single-effect multistage flash distillation with a recycle operation. The main reasons for using a recycle stream are to increase the total heat capacity of the flashing brine and the percentage conversion of the brine feed into fresh water (5, 6). Due to the high latent heat of vaporization of water and the low heat capacity of the aqueous solution, the solution cools off considerably when only a small fraction of the solution is flash evaporated. In the flash distillation process, the highest flashing temperature is limited by the scale formation problem and the lowest temperature is limited by the temperature of the seawater which is used as the coolant. The percentage conversion of a flash distillation process without recycle, which is operated within the temperature range mentioned above, is less than 20% (7, 8). Since the saline water feed stream has to be pumped and pretreated, a low percentage conversion of feed water into fresh water will result in a poor overall economy for the process.

A MEMS process consists of several SEMS plants with recycles connected in series. Each SEMS system is considered as an effect. W. R. Williamson et. al. (7) have summarized the advantages of the MEMS system as compared to the SEMS system as follows:

(a). Reduces total feed treatment cost by 50%.

(b). Reduces heat transfer surface by at least 20%.

(c). Allows for more stages at the hot end of the plant and fewer stages at the cold end of the plant.

Thus, a MEMS system lends itself to better control of the operating variables so that a lower water cost can be achieved.

The MEMS process is described in detail in Chapter 2. In Chapter 3 a mathematical model which fairly accurately describes the process is

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developed. Each effect is first assumed to consist of an infinite number of stages (infinite stage operation) and differential equations are set up to obtain the idealized performance equations. Since an actual plant consists of a finite number of stages (finite stage operation), appropriate correction terms are added to the idealized performance equations. The capital and operating cost equations are set up in Chapter 4.

The discrete maximum principle combined with a search technique is used to optimize the MEMS system. Two search techniques are used in this study: the parametric search and the Simplex method. The optimization procedure and numerical results are illustrated and the comparison of the two methods is discussed in Chapter 5. The final optimal policy of the system and the capital and operating costs allocation are given in Chapter 6. The computer program of each method and the sample results are listed in the Appendix.

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CHAPTER 2

PROCESS DESCRIPTION

Figure 1 presents a simplified process flow diagram of a three-effect multistage flash system and Figure 2 depicts a typical effect, the n-th effect, of the system. The critical locations of the system are denoted by numbers, n', n", and n, which divide the system into various sections, namely sections H-n, MR-n, HR-n, and R-n. The n-th effect consists of preheater H-n, mixing section MR-n, heat recovery section HR-n, and heat rejection section R-n. From Figure 2 it is easily seen that the preheater of the n-th effect, H-n, coincides with the heat rejection section, R-(n-1), of the (n-1)-th effect.

F, L, and R_n represent respectively the flow rate of the feed brine, flashing brine, and recycle brine in the n-th effect. W_n represents the condensate produced in the n-th effect. The feed brine and the recycle brine together are referred to as the non-flashing brine stream. T_f, T_j, and T_c represent respectively the temperature of the flashing brine, non-flashing brine, and condensate. The subscript is used to indicate the location. For example, $(T_f)_1$ and $(T_j)_{n'}$ are respectively the temperature of the flashing brine at location 1 and temperature of the non-flashing brine at location n'.

In Figure 1, the seawater feed is heated in section R-3 and then degasified to remove CO_2 and other dissolved gases. After being heated successively in sections HR-3, H-3, HR-2, H-2, and HR-1, it is mixed with the recycle brine R_1 to form a brine stream which is heated in the brine heater, H-1, and then introduced into the first effect as the flashing brine $(L)_{1'}$.

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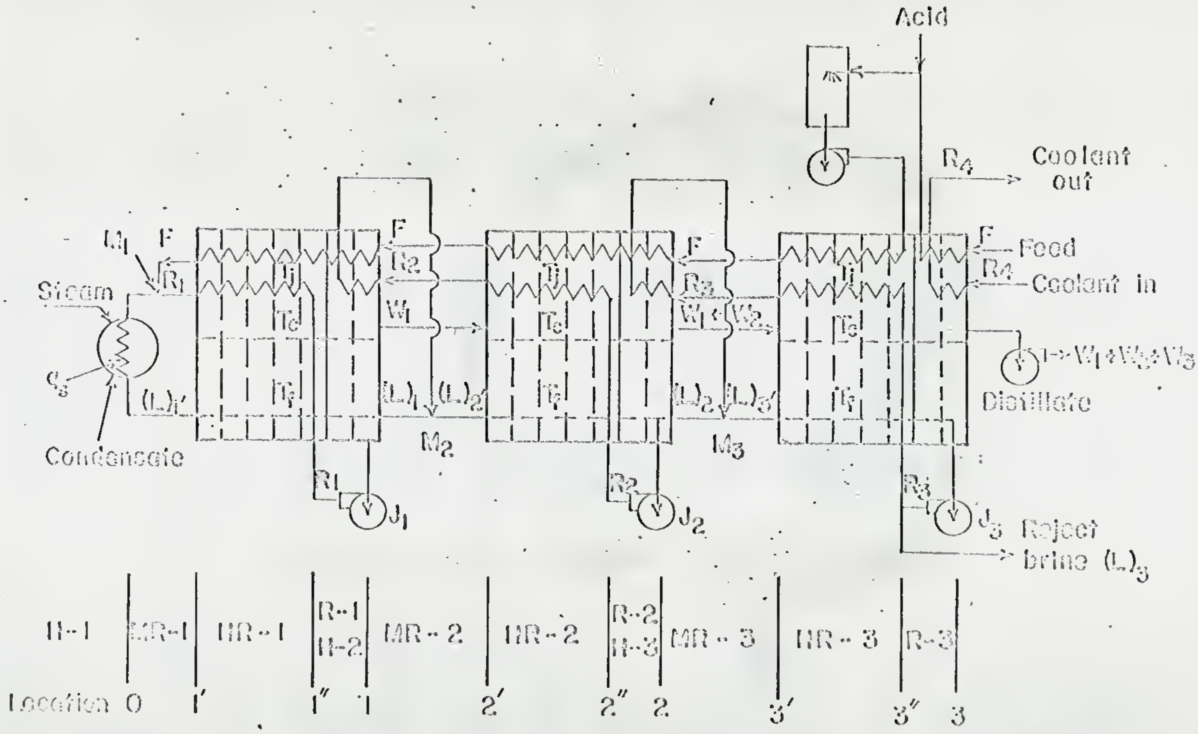
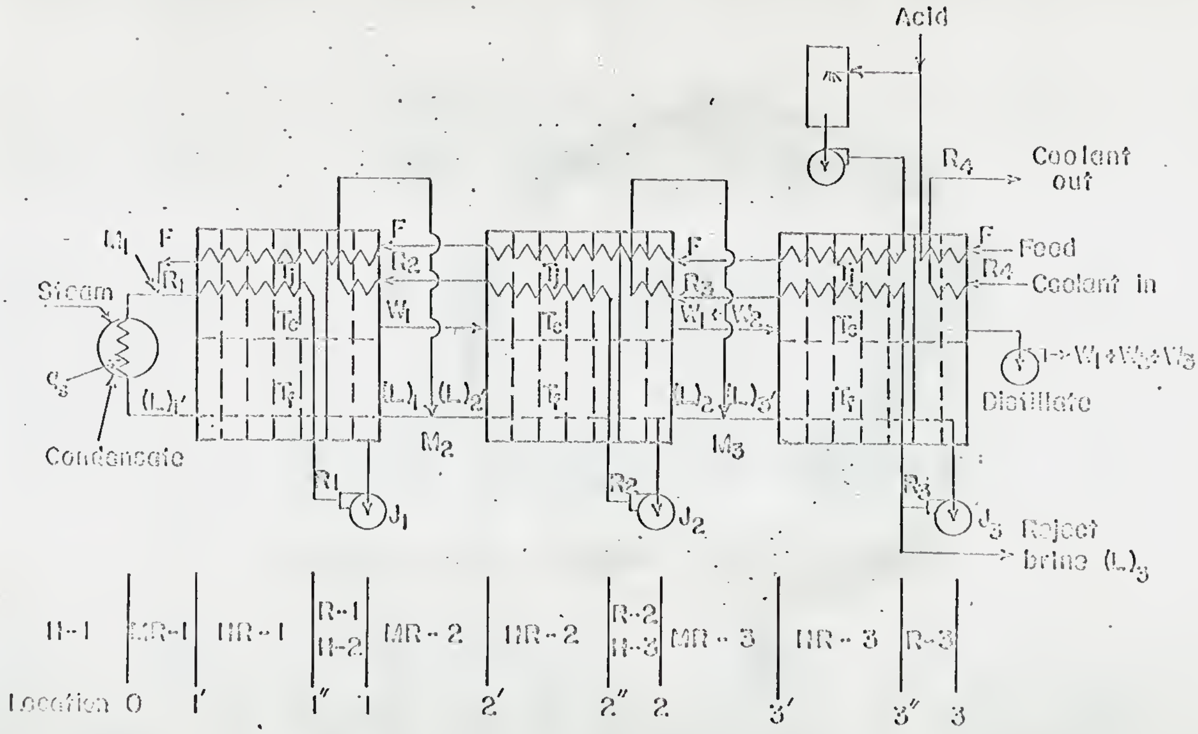


Fig. 1. Multi-effect multistage flash distillation system.



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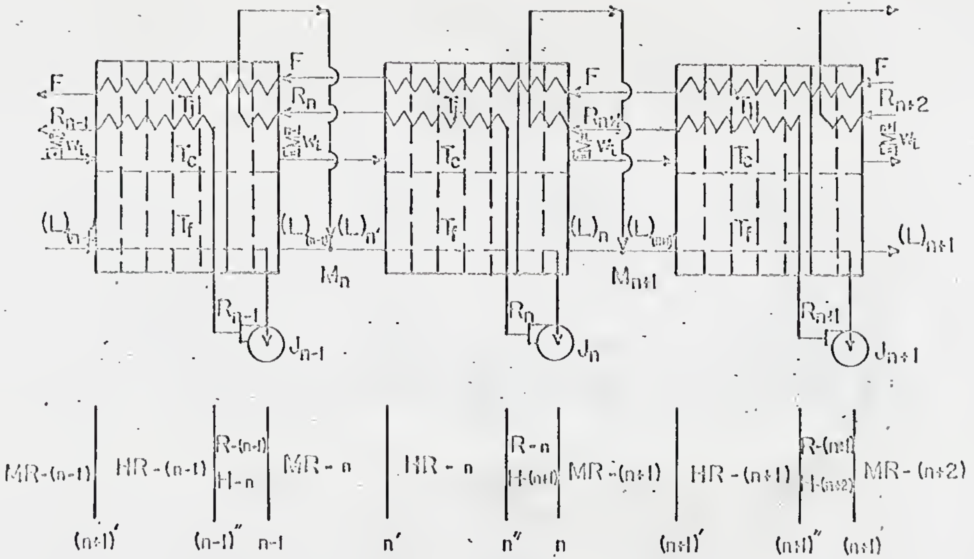

Fig. 2. The n-th effect of the multi-effect multi-stage flash distillation system.

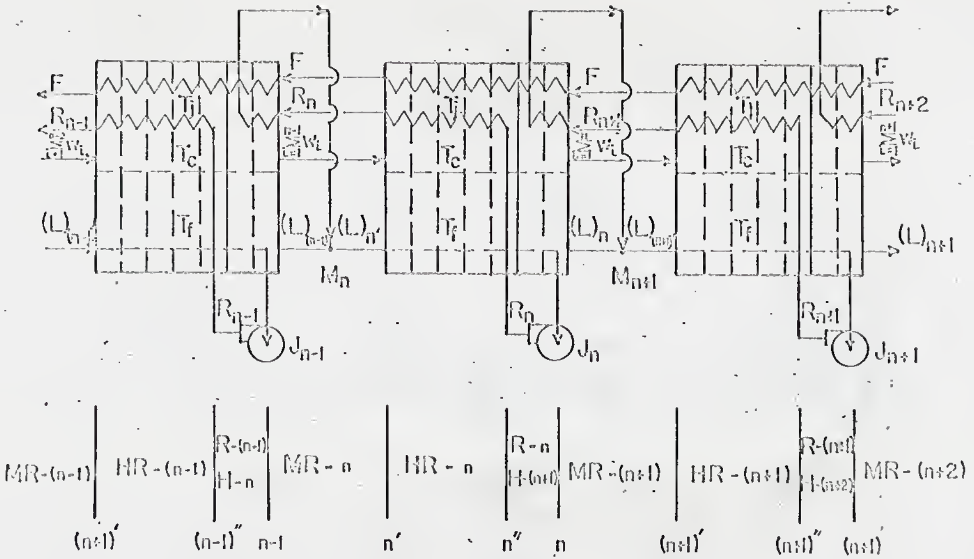

Fig. 2. The n-th effect of the multi-effect multi-stage flash distillation system.

As is shown in Figure 2, the flashing brine at location n is divided into two streams: one stream, $(L)_n$, is fed into the (n+1)-th effect and the other stream, R_n, is recirculated by the recycle pump, J_n, heated in sections HR-n and H-n, and then mixed with the brine stream $(L)_{n-1}$ at the mixing point, M_n, and then the combined stream is introduced into the n-th effect as the brine stream, $(L)_{n'}$.

The feed brine and the recycle brine are heated in each stage by the water vapor evaporated from the flashing brine in that stage. It is possible to arrange the flow system so that the temperatures of the feed brine and the recycle brine are equal at any location. In the following discussion, such an arrangement is assumed. As has been described, the feed brine and the recycle brine together are referred to as the non-flashing brine and its temperature is denoted by T_j. The recycle brine, R_n, which is a part of the flashing brine at location n, is introduced into the condensing chamber at location n" where it becomes a part of the non-flashing brine stream. Thus, the following relation should hold.

$$(T_j)_{n''} = (T_f)_n \quad , \quad n = 1, 2, 3.$$

Therefore, the two brine streams, R_{n+1} and $(L)_n$, which are mixed at the mixing point, M_{n+1}, are at the same temperature but are at different concentration levels. Because of the rather limited concentration range of approximately from 3.5% to 7% encountered in the process, the heat of mixing due to concentration difference is assumed negligible. From this assumption, one can see that the temperature of the brine stream before and after mixing must remain unchanged, i.e.,

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$$(T_f)_n = (T_f)_{(n+1)} \quad , \quad n = 1, 2. \tag{2}$$

A stage within each effect consists of a flashing chamber and a condensing chamber and a demister which separates the two chambers. Each stage is maintained at a lower pressure than the preceding one. Brine flows from stage to stage, giving up additional vapor as the pressure drops; the vapor then passes through the demister to the condensing chamber, where it is condensed to heat the non-flashing brine.

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CHAPTER 3

PROCESS ANALYSIS

3-1. Outline of Process Analysis

Quantitative relations among the operating variables are derived in the following sections. The performance of a MEMS system is characterized by the temperature - composition diagram for the (n-1)th and the n-th effects in Figure 3. The general approach is to obtain idealized performance equations by assuming infinite stage operation in each effect and applying correction terms for the finiteness of the number of stages.

The lines, a-b-c, and d-e-f, show how the temperature of the flashing brine, T_f, decreases as the concentration of the brine, C_f, increases in an infinite stage operation in the (n-1)-th effect and n-th effect, respectively. The relations representing these lines are derived in section 3-5. The concentration gaps, between (n-1) and n', n and (n+1), are caused by mixing of brines due to recirculation in the n-th and (n+1)-th effects, respectively. The stepped lines along the lines, a-b-c and d-e-f, represent the temperature of the flashing brine in the (n-1)-th, and n-th effects respectively in an actual process where the number of stages in each effect is finite.

The lines, a'-b'-c', and d'-e'-f', show the relations between the condensate temperature, T_c, and the flash brine composition, C_f, at various locations in the system for an infinite stage operation. The stepped lines along a'-b'-c' and d'-e'-f' again represent an actual finite stage operation.

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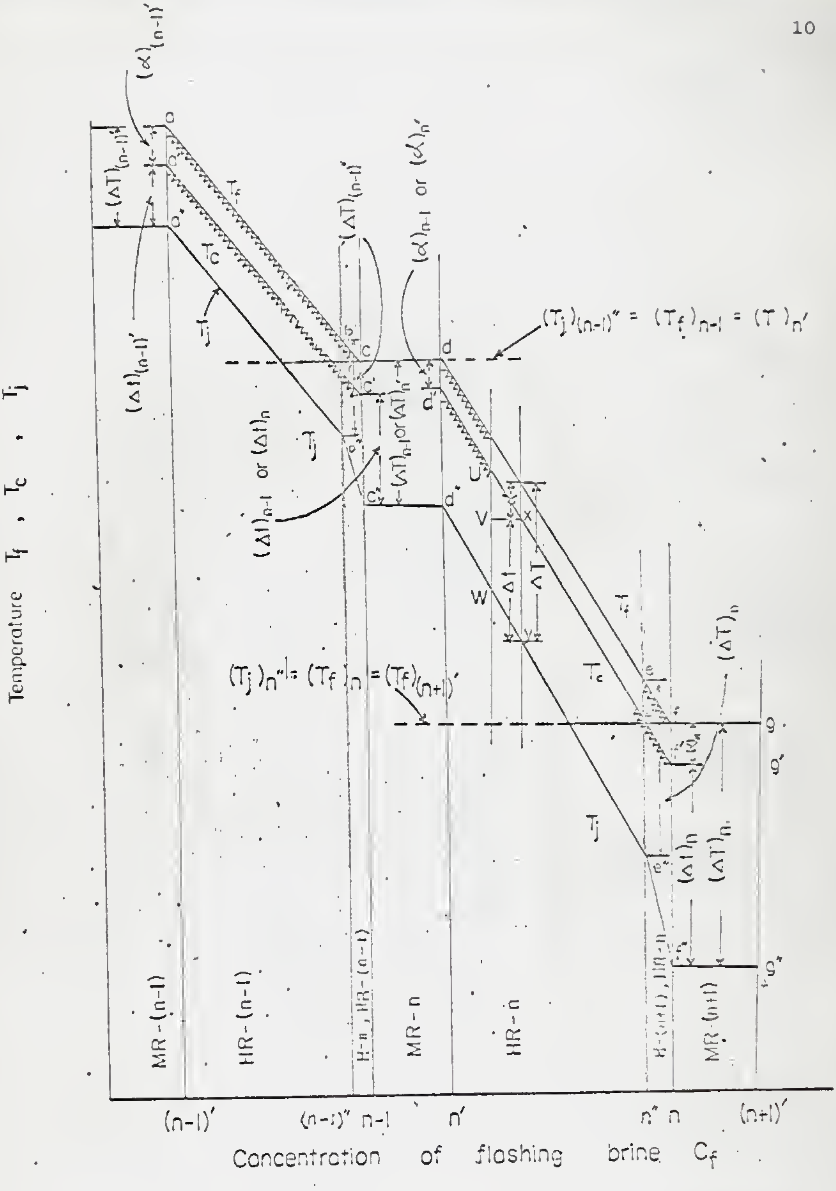

Fig. 3. Temperatures T_f , T_c , T_j vs concentration of flashing brine C_f (schematic).

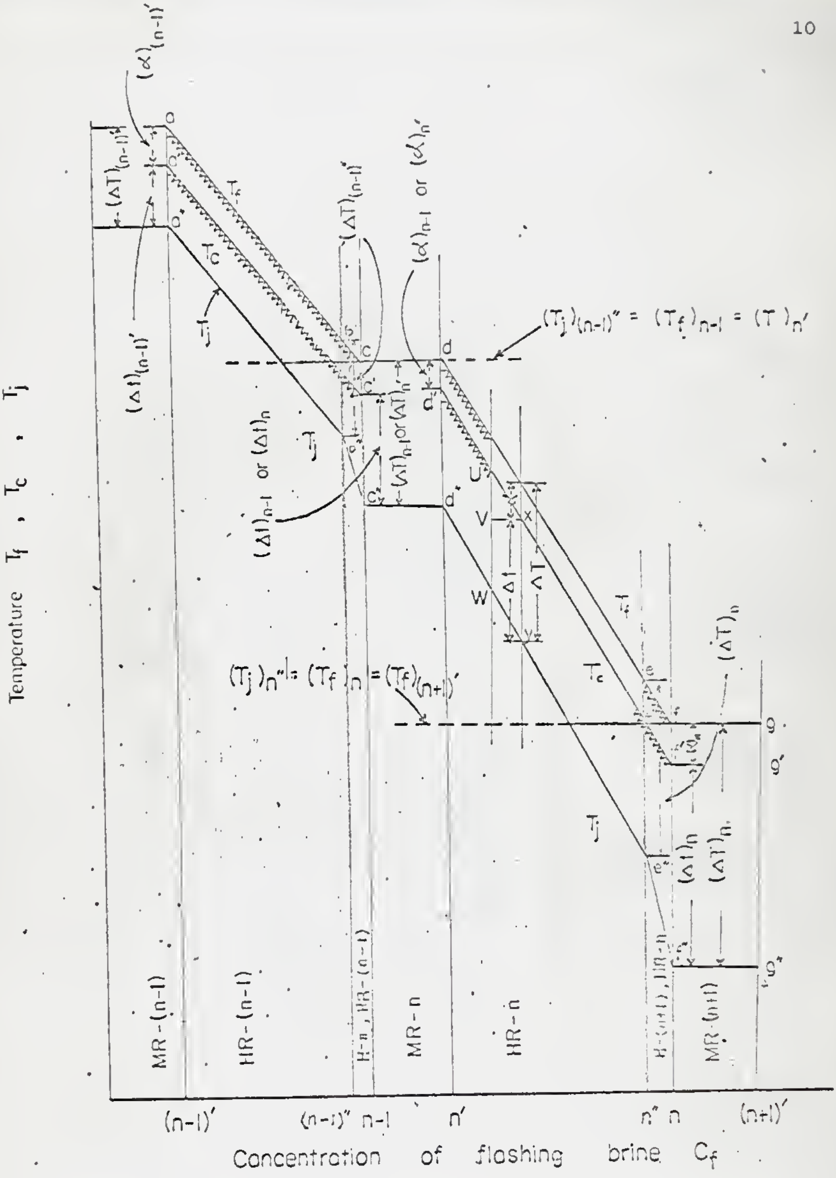

Fig. 3. Temperatures T_f , T_c , T_j vs concentration of flashing brine C_f (schematic).

The vertical distance between the two sets of lines described in the last two paragraphs represents $(T_f - T_c)$ at various locations in the system. This difference is due to the boiling point elevation of the flashing brine and the pressure difference across the demister at each location. This difference will be denoted by α. The magnitude of α varies throughout an effect; this is mainly due to the varying composition, and consequently the varying boiling point elevation. The average value of α in the n-th effect is denoted by α_n and is derived in section 3-6.

Similarly, the lines a"-b"-c" and d"-e"-f" in Figure 3 show the relation between non-flashing brine temperature, T_j, and flashing brine composition, C_t. ΔT is used to represent the temperature difference $(T_f - T_j)$. As illustrated in the figure, ΔT, in an infinite stage operation is nearly constant within a heat recovery section. However, it is not constant within a heat rejection section. For example, ΔT varies gradually from $(\Delta T)_{n''}$ to $(\Delta T)_n$ in the heat rejection section R-n. These items are explained further in section 3-8. The average ΔT in the n-th effect, ΔT_n, is derived in section 3-9.

The heat loads in the brine heater and the n-th effect are derived in section 3-4. Δt is used to denote the temperature difference required for the heat transfer in the various sections. The average values of Δt in the brine heater and the n-th effect are calculated by the relations derived in section 3-10. From these, the relations for the heat transfer area requirements are derived in section 3-11. The pumping head required for each of the circulating pumps is derived in section 3-12.

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3-2. Flow Rates of Flashing Brine, Recycle Brine and Condensate Streams.

The notation representing the various fluid streams has been defined in Chapter II. The recycle ratio in the n-th effect, r_n, is defined by

$$r_n = \frac{R_n}{(L)_{n-1}}, \qquad n = 1, 2, 3, \tag{3}$$

where

$$(L)_0 = F.$$

The flow rate of the brine stream leaving the n-th effect, $(L)_n$ is related to its concentration, $(C_f)_n$, by the following equation,

$$(L)_n = F \frac{C_F}{(C_f)_n}, \qquad n = 1, 2, 3, \tag{4}$$

where C_F is the salt concentration in the feed. Therefore, by combining equations (3) and (4), the flow rate of the recycle brine stream can be written as

$$R_n = r_n F \frac{C_F}{(C_f)_{n-1}}, \qquad n = 1, 2, 3. \tag{5}$$

Note that

$$(C_f)_0 = C_F.$$

The flow rate of the brine stream entering the n-th effect is given by

$$(L)_{n'} = (L)_{n-1} + R_n = F \frac{C_F}{(C_f)_{n-1}} (1+r_n), \quad n = 1, 2, 3. \tag{6}$$

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From the overall material balance over the n-th effect, we obtain the following equation for the flow rate of the condensate produced in the n th effect.

$$W_n = F\left\{\frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n}\right\} \quad , \quad n = 1, 2, 3. \tag{7}$$

Therefore, the total water production, $\sum W_n$, is

$$\Sigma W = \sum_{n=1}^{3} W_n = F\left\{1 - \frac{C_F}{(C_f)_3}\right\} \quad . \tag{8}$$

3-3. Mixing of the Recycle Brine Stream with the Flashing Brine Stream.

As mentioned previously, the brine streams having different compositions are mixed isothermally at mixing points. By making a salt material balance at the mixing point, M_n, as shown in Figure 2, we obtain

$$(L)_{n-1} \; (C_f)_{n-1} + R_n \; (C_f)_n = (L)_{n'} \; (C_f)_{n'}$$

Substituting equations (4), (5), and (6) into the foregoing equation yields

$$(C_f)_{n'} = \frac{(C_f)_{n-1} + r_n (C_f)_n}{1 + r_n} \quad , \quad n = 1, 2, 3. \tag{9}$$

3-4. The Heat Loads in the Brine Heater and the n-th Effect.

The heat load, q_s, in the brine heater, is a very important operating variable. A large value of q_s gives rise to an increased steam cost. But it also gives rise to a large temperature difference Δt for heat transfer and consequently a low plant cost.

From the overall material balance over the n-th effect, we obtain the following equation for the flow rate of the condensate produced in the n th effect.

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3-4. The Heat Loads in the Brine Heater and the n-th Effect.

The heat load, q_s, in the brine heater, is a very important operating variable. A large value of q_s gives rise to an increased steam cost. But it also gives rise to a large temperature difference Δt for heat transfer and consequently a low plant cost.

If the heat load due to flashing of the condensate stream is neglected, the heat load in the n-th effect can be approximated by the latent heat required for condensate production in the n-th effect. Therefore, we have

$$q_n = W_n \lambda = F\left\{\frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n}\right\}\lambda , \quad n = 1, 2, 3, \qquad (10)$$

where λ is the latent heat of flashing brine.

3-5. Temperature and Composition of the Flashing Brine, T_f vs. C_f.

In Figure 4, which represents a flashing chamber of an infinite stage system, L, C_f, T_f and h_f are respectively the quantity, concentration, temperature and unit enthalpy of the flashing brine. Let dV be the quantity of water vapor evaporated, and let H_v be the unit enthalpy of the vapor. A total material balance gives

$$L = L + dL + dV$$

or

$$dL = - dV. \qquad (11)$$

A salt balance gives

$$L\ C_f = (L + dL)\ (C_f + dC_f),$$

Neglecting the term $dLdC_f$ in this equation yields

$$\frac{dL}{L} = - \frac{dC_f}{C_f} . \qquad (12)$$

The enthalpy balance is

$$L\ h_f = (L + dL)(h_f + dh_f) + H_v dV.$$

If the heat load due to flashing of the condensate stream is neglected, the heat load in the n-th effect can be approximated by the latent heat required for condensate production in the n-th effect. Therefore, we have

$$q_n = W_n \lambda = F\left\{\frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n}\right\}\lambda \quad , \quad n = 1, 2, 3, \qquad (10)$$

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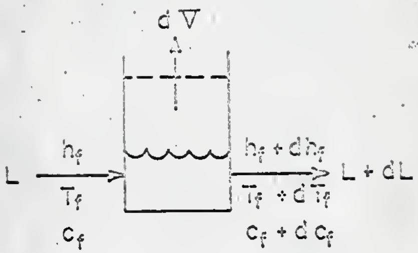

Fig. 4. The flashing chamber of a stage in the infinite stage system.

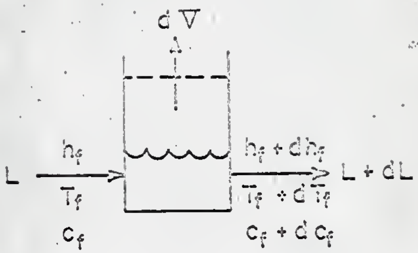

Fig. 4. The flashing chamber of a stage in the infinite stage system.

Neglecting the term $dLdh_f$ and substituting equation (11) into the above equation yields

$$L\, dh_f = (H_v - h_f)\, dL.$$

Since $dh_f = C_p\, dT_f$ and $H_v - h_f$ can be approximated by the latent heat of vaporization, λ, the above equation becomes

$$\frac{C_p}{\lambda}\, dT_f = \frac{dL}{L} .$$

By substituting equation (12) into the foregoing equation, we obtain

$$\frac{C_p}{\lambda}\, dT_f = - \frac{dC_f}{C_f} . \qquad (13)$$

Assuming that C_p/λ is constant and integrating the above equation between locations n' and n, we have

$$\ln \frac{(C_f)_n}{(C_f)_{n'}} = \frac{C_p}{\lambda} \left\{ (T_f)_{n'} - (T_f)_n \right\} . \qquad (14)$$

Substituting equation (9) into this equation and noting that

$$(T_f)_{n'} = (T_f)_{n-1},$$

we obtain

$$\ln (C_f)_n = \ln \frac{(C_f)_{n-1} + r_n (C_f)_n}{1 + r_n} + \frac{C_p}{\lambda} \left\{ (T_f)_{n-1} - (T_f)_{n'} \right\} .$$

$$n = 1, 2, 3. \qquad (15)$$

This is the equation of the straight line, d-e-f, in Figure 3.

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Substituting equation (9) into this equation and noting that

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$$n = 1, 2, 3. \qquad (15)$$

This is the equation of the straight line, d-e-f, in Figure 3.

3-6. Average α in the n-th Effect, α_n .

α is defined as the difference in temperature of the flashing brine and the condensate in a state. α_n is the average value of α in the n-th effect. The value of α in a stage depends on the boiling point elevation of the brine and the drop in condensation temperature due to the demister pressure drop. Due to lack of information, the drop in condensing temperature due to the demister pressure drop was assumed to be 1^oF in each effect.

Figure 5 shows the boiling point elevation of the brine solution as a function of its composition at constant temperature. The average value of the boiling point elevation in each effect was evaluated at the average temperature in the corresponding effect. The average temperatures in the first, second and third effects are assumed to be 225^oF, 175^oF and 125^oF, respectively. For further simplification, the average value of the boiling point elevation in each effect was calculated from the slope of each curve at the appropriate concentration ranges instead of the curve itself.

Therefore, α_n can be expressed as functions of the average brine composition in the n-th effect by the following equations,

$$\alpha_1 = 1.01 + \frac{1}{0.03} \quad \frac{(C_f)_{1'} + (C_f)_1}{2}$$

$$= 1.01 + \frac{1}{0.03} \left\{ \frac{\frac{C_F+r_1(C_f)_1}{1 + r_1} + (C_f)_1}{2} \right\} . \qquad (16)$$

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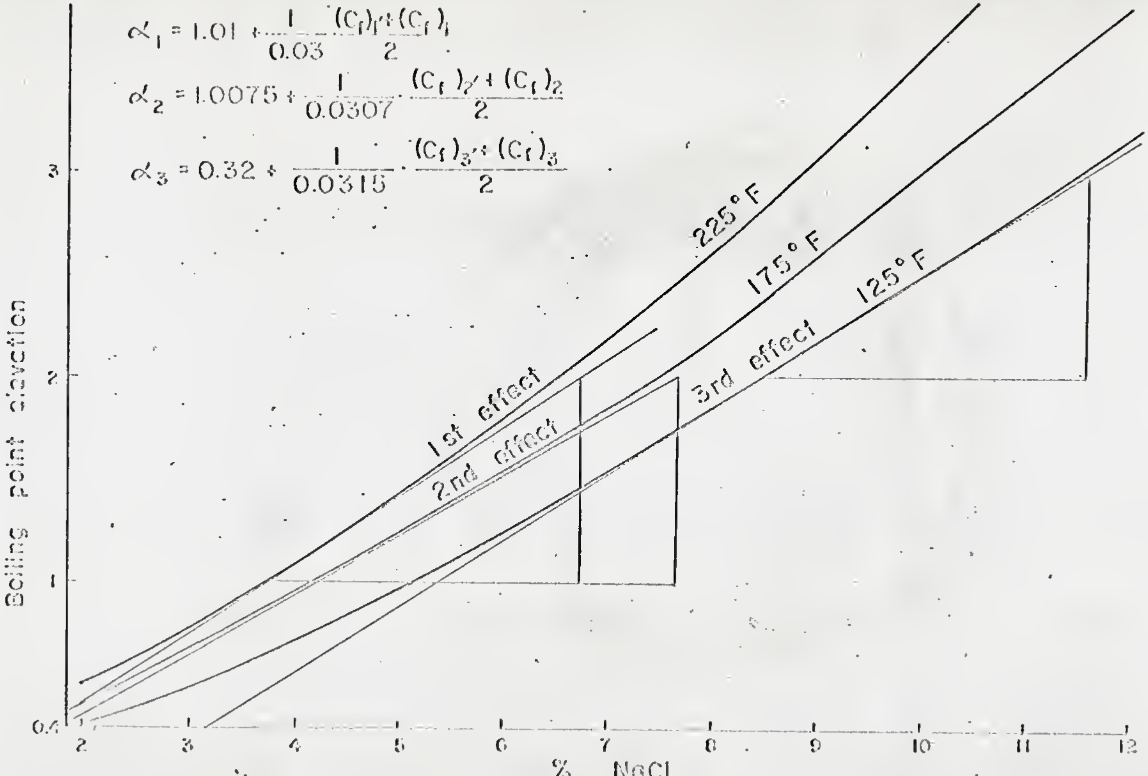

Fig. 5. Boiling point elevation vs % NaCl in salt solution at 125°F, 175°F and 225°F.

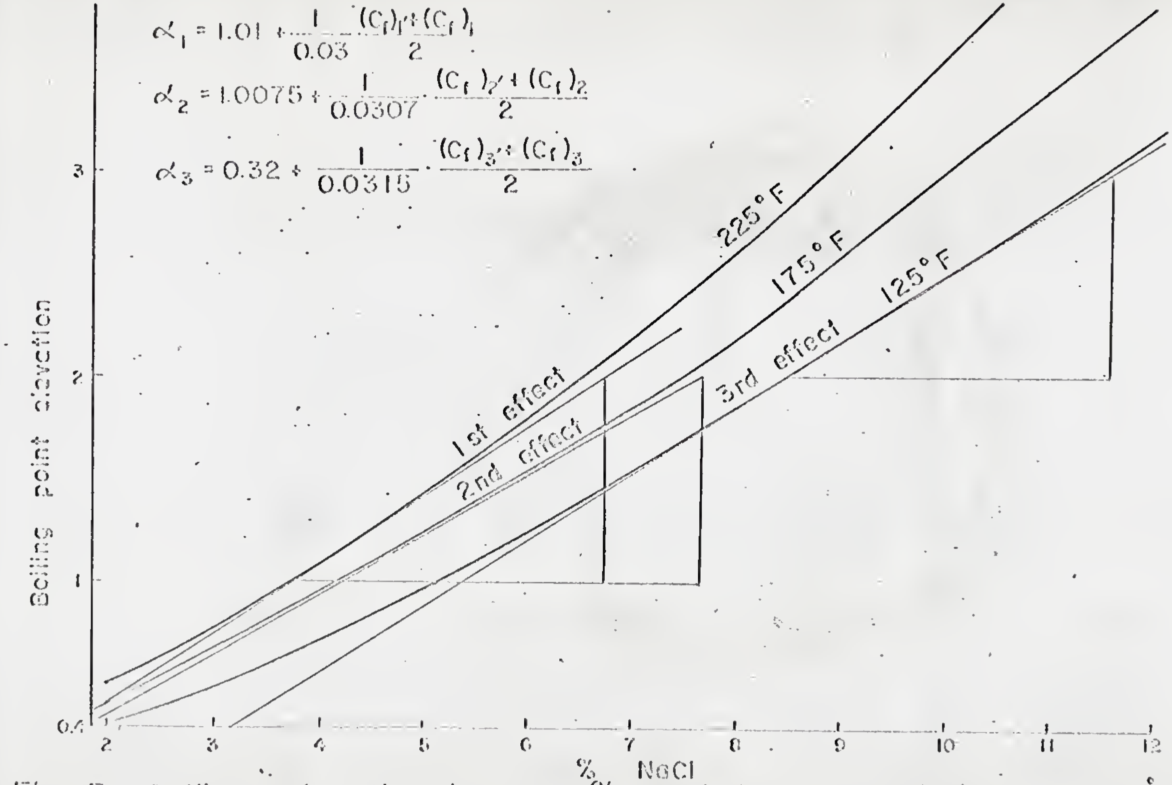

Fig. 5. Boiling point elevation vs % NaCl in salt solution at 125°F, 175°F and 225°F.

$$\alpha_2 = 1.0075 + \frac{1}{0.0347} \; \frac{(C_f)_{2'} + (C_f)_2}{2}$$

$$= 1.0075 + \frac{1}{0.0347} \; \frac{\dfrac{(C_f)_1 + r_2 (C_f)_2}{1 + r_2} + (C_f)_2}{2} \qquad (17)$$

$$\alpha_3 = 0.32 + \frac{1}{0.0375} \; \frac{(C_f)_{3'} + (C_f)_3}{2}$$

$$= 0.32 + \frac{1}{0.0375} \; \frac{\dfrac{(C_f)_2 + r_3 (C_f)_3}{1 + r_3} + (C_f)_3}{2} \; . \qquad (18)$$

3-7. The Flow Rate of the Cooling Water, R_4.

An enthalpy balance around the whole system gives

$$q_s + (F + R_4) C_p (T_j)_{3''} = R_4 C_p (T_j)_{3''} + W_f C_p (T_c)_3 + (L)_3 C_p (T_f)_3 \; .$$

Since

$$(T_j)_{3''} = (T_f)_3 \; , \; (T_c)_3 = (T_f)_3 - \alpha_3 \; , \text{ and } F = W_f + (L)_3 \; ,$$

the above equation can be solved for the cooling water flow rate as

$$R_4 = \frac{\dfrac{q_s}{C_p} + \Sigma W_n \alpha_3}{(T_f)_3 - (T_j)_3} - F \qquad (19)$$

$$\alpha_2 = 1.0075 + \frac{1}{0.0347} \; \frac{(C_f)_{2'} + (C_f)_2}{2}$$

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Since

$$(T_j)_{3''} = (T_f)_3 \; , \; (T_c)_3 = (T_f)_3 - \alpha_3 \; , \text{ and } F = W_f + (L)_3 \; ,$$

the above equation can be solved for the cooling water flow rate as

$$R_4 = \frac{\dfrac{q_s}{C_p} + \Sigma W_n \alpha_3}{(T_f)_3 - (T_j)_3} - F \tag{19}$$

From the foregoing equation we obtain

$$\frac{R_4}{W_n} = \frac{\frac{q_s}{F}\,\frac{1}{C_p}\,\frac{1}{1 - \frac{C_F}{(C_f)_3}} + \alpha_3}{(T_f)_3 - (T_j)_3} - \frac{1}{1 - \frac{C_F}{(C_f)_3}} \quad . \qquad (20)$$

3-8. The Temperature Difference Between the Flashing Brine and Non-Non-Flashing Brine, ΔT, in Sections HR-n, and R-n.

A) ΔT in Section HR-n.

In Figure 6 a whole stage in the infinite stage system is taken for analysis. The brine feed stream, F, and the recycle brine stream, R_n, are introduced to the stage from the right and leave from the left. The condensate, W, and the flashing brine stream enter the stage from the left and leave from the right. The temperature, unit enthalpy, and quantity of each stream per hour are denoted in the figure.

By making an energy balance around this stage, the following relation was obtained.

$$(F + R_n)(h_j + dh_j) + W\,h_c + L\,h_f$$

$$= (F + R_n)h_j + (W + dW)(h_c + dh_c) + (L + dL)(h_f + dh_f).$$

Since $dh_c = dh_f$, $- dL = dW$ and $L + W = F + R_n$, the above equation becomes

From the foregoing equation we obtain

$$\frac{R_4}{W_n} = \frac{\dfrac{q_s}{F}\dfrac{1}{C_p}\dfrac{1}{1-\dfrac{C_F}{(C_f)_3}} + \alpha_3}{(T_f)_3 - (T_j)_3} - \frac{1}{1-\dfrac{C_F}{(C_f)_3}} . \qquad (20)$$

3-8. The Temperature Difference Between the Flashing Brine and Non-Non-Flashing Brine, ΔT, in Sections HR-n, and R-n.

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$$= (F + R_n)h_j + (W + dW)(h_c + dh_c) + (L + dL)(h_f + dh_f).$$

Since $dh_c = dh_f$, $- dL = dW$ and $L + W = F + R_n$, the above equation becomes

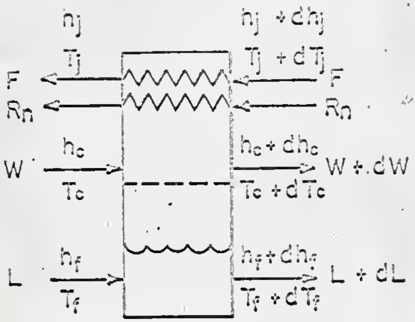

Fig. 6. A stage in the infinite stage system.

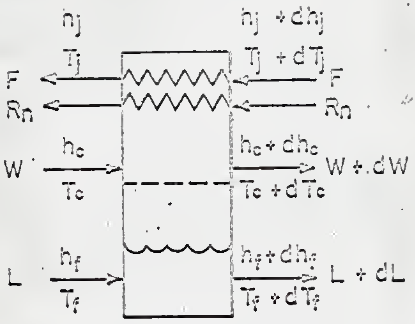

Fig. 6. A stage in the infinite stage system.

$$
\begin{aligned}
(F + R_n)dh_j &= W\,dh_c + h_c dW + L dh_f + h_f\,dL \\
&= (L + W)dh_f - (h_f - h_c)dW \\
&= (F + R_n)dh_f - (h_f - h_c)dW. \qquad (21)
\end{aligned}
$$

Since the feed brine and the recycle brine receive the heat of condensation of the water vapor, we have

$$- (F + R_n)dh_j = \lambda\,dW$$

or

$$- dW = \frac{(F + R_n)dh_j}{\lambda}\,. \qquad (22)$$

Substituting equation (22) into equation (21) yields

$$(F + R_n)dh_j = (F + R_n)dh_f + \frac{(h_f - h_c)(F + R_n)dh_j}{\lambda}. \qquad (23)$$

On rearranging, we obtain

$$\left(1 - \frac{h_f - h_c}{\lambda}\right) dh_j = dh_f. \qquad (24)$$

Since

$$\frac{h_f - h_c}{\lambda} < < 1,$$

equation (24) can be approximated by

$$dh_j = dh_f \qquad (25)$$

or

$$dT_j = dT_f. \qquad (26)$$

$$
\begin{aligned}
(F + R_n)dh_j &= W\,dh_c + h_c dW + L dh_f + h_f\,dL \\
&= (L + W)dh_f - (h_f - h_c)dW \\
&= (F + R_n)dh_f - (h_f - h_c)dW. \qquad (21)
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equation (24) can be approximated by

$$dh_j = dh_f \qquad (25)$$

or

$$dT_j = dT_f. \qquad (26)$$

Integrating this between location n' and n" and rearranging gives

$$(\Delta T)_{n'} = (T_f)_{n'} - (T_j)_{n'} = (T_f)_{n''} - (T_j)_{n''} = (\Delta T)_{n''} \quad (27)$$

This derivation leads to the conclusion that ΔT is nearly constant in the heat recovery sections.

B) ΔT in Section R-n.

In the heat rejection section, a derivation similar to that described above leads to the following result,

$$(F + R_n)dT_f = (F + R_{n+1})dT_j. \quad (28)$$

Since both $F + R_n$ and $F + R_{n+1}$ are constant, the equation can be integrated between location n" and n to give

$$(F + R_n) \left\{ (T_f)_{n''} - (T_f)_n \right\} = (F + R_{N+1}) \left\{ (T_j)_{n''} - (T_j)_n \right\}.$$

Since $(T_f)_n = (T_j)_{n''}$, the above equation becomes

$$(F + R_n) \left\{ (T_f)_{n''} - (T_j)_{n''} \right\} = (F + R_{n+1}) \left\{ (T_f)_n - (T_j)_n \right\} \quad (29)$$

or

$$(F + R_n)(\Delta T)_{n''} = (F + R_{n+1}) (\Delta T)_n . \quad (30)$$

Therefore the temperature difference varies gradually from

$(\Delta T)_{n''}$ to $(\Delta T)_n$, as is shown in Figure 3.

3-9. Average ΔT in the n-th Effect, ΔT_n

Integrating this between location n' and n" and rearranging gives

$$(\Delta T)_{n'} = (T_f)_{n'} - (T_j)_{n'} = (T_f)_{n''} - (T_j)_{n''} = (\Delta T)_{n''} \quad (27)$$

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$$(F + R_n) \left\{ (T_f)_{n''} - (T_f)_n \right\} = (F + R_{N+1}) \left\{ (T_j)_{n''} - (T_j)_n \right\}.$$

Since $(T_f)_n = (T_j)_{n''}$, the above equation becomes

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or

$$(F + R_n)(\Delta T)_{n''} = (F + R_{n+1}) (\Delta T)_n . \quad (30)$$

Therefore the temperature difference varies gradually from $(\Delta T)_{n''}$ to $(\Delta T)_n$, as is shown in Figure 3.

3-9. Average ΔT in the n-th Effect, ΔT_n

It is known from the last section that ΔT is constant in the heat recovery section of the n-th effect, that is, $(\Delta T)_{n'} = (\Delta T)_{n''}$. In the heat rejection section, however, ΔT changes slightly from $(\Delta T)_{n''}$ to $(\Delta T)_n$ according to equation (30). However, since the heat recovery section contains as large a number of stages as the heat rejection section does, we may reasonably assume that the ΔT in the heat recovery section is the average ΔT in the effect. Therefore, we have

$$\Delta T_n = (\Delta T)_{n'} = (\Delta T)_{n''} \tag{31}$$

By applying an enthalpy balance between locations 0 and n', we obtain

$$q_s + (F + R_n)\, C_p (T_j)_{n'} = \left(\sum_{i=1}^{n=1} W_i\right) C_p (T_c)_{n'} + (L)_{n'} C_p (T_f)_{n'}$$

Substituting equations (5), (6), and (8) into the above equation and rearranging yields

$$(T_f)_{n'} - (T_j)_{n'} \left(1 + \frac{r_n\, C_F}{(C_f)_{n-1}}\right) = \frac{q_s}{F}\frac{1}{C_p} + \left(1 - \frac{C_F}{(C_f)_{n-1}}\right)\left[(T_f)_{n'} - (T_c)_{n'}\right].$$

Since $(T_f)_{n'} - (T_c)_{n'} = \alpha_n$, and $(T_f)_{n'} - (T_j)_{n'} = \Delta T_n$, the foregoing equation becomes

$$\Delta T_n = \frac{\dfrac{q_s}{F}\dfrac{1}{C_p} + \left(1 - \dfrac{C_F}{(C_f)_{n-1}}\right)\alpha_n}{1 + \dfrac{r_n\, C_F}{(C_f)_{n-1}}}, \quad n = 1, 2, 3. \tag{32}$$

3-10. The Effective Δt for Heat Transfer in the Brine Heater, Δt_0, and in the n-th Effect, Δt_n.

A) Δt in the brine heater, Δt_0

If we let T_s be the steam temperature, then

Δt at the inlet $= T_s - (T_j)_1$,

Δt at the outlet $= T_s - (T_f)_1$,

It is known from the last section that ΔT is constant in the heat recovery section of the n-th effect, that is, $(\Delta T)_{n'} = (\Delta T)_{n''}$. In the heat rejection section, however, ΔT changes slightly from $(\Delta T)_{n''}$ to $(\Delta T)_n$ according to equation (30). However, since the heat recovery section contains as large a number of stages as the heat rejection section does, we may reasonably assume that the ΔT in the heat recovery section is the average ΔT in the effect. Therefore, we have

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Substituting equations (5), (6), and (8) into the above equation and rearranging yields

$$(T_f)_{n'} - (T_j)_{n'} \left(1 + \frac{r_n\, C_F}{(C_f)_{n-1}}\right) = \frac{q_s}{F} \frac{1}{C_p} + \left(1 - \frac{C_F}{(C_f)_{n-1}}\right) \left[(T_f)_{n'} - (T_c)_{n'}\right].$$

Since $(T_f)_{n'} - (T_c)_{n'} = \alpha_n$, and $(T_f)_{n'} - (T_j)_{n'} = \Delta T_n$, the foregoing equation becomes

$$\Delta T_n = \frac{\frac{q_s}{F} \frac{1}{C_p} + \left(1 - \frac{C_F}{(C_f)_{n-1}}\right) \alpha_n}{1 + \frac{r_n\, C_F}{(C_f)_{n-1}}}, \quad n = 1, 2, 3. \tag{32}$$

3-10. The Effective Δt for Heat Transfer in the Brine Heater, Δt_0, and in the n-th Effect, Δt_n.

A) Δt in the brine heater, Δt_0

If we let T_s be the steam temperature, then

Δt at the inlet $= T_s - (T_j)_1$,

Δt at the outlet $= T_s - (T_f)_1$,

and the average Δt for the heat transfer in the brine heater is

$$\Delta t_0 = T_s - \frac{1}{2}\left[(T_j)_{1'} + (T_f)_{1'}\right] .$$

Since

$$\Delta T_1 = (T_f)_{1'} - (T_j)_{1'} ,$$

the above equation can be rearranged to give

$$\Delta t_0 = T_s - (T_f)_{1'} + \frac{1}{2}\Delta T_1$$

$$= T_s - (T_f)_0 + \frac{1}{2}\,\frac{q_s/F}{C_p'(1 + r_1)} \qquad (33)$$

where

$$(T_f)_0 = (T_f)_{1'} .$$

The maximum value of $(T_f)_0$ must generally be limited in order to control scale formation. In this study, the values of T_s and $(T_f)_0$ are assumed to be fixed.

B). Δt in the n-th Effect, Δt_n .

Referring to a stage in section HR-n of Figure 3, it can be seen that the effective Δt for heat transfer in the infinite stage operation is given by

$$\frac{uw + xy}{2} = uw = xy .$$

For N stage operation, the effective Δt becomes

$$\frac{vw + xy}{2}$$

and the average Δt for the heat transfer in the brine heater is

$$\Delta t_0 = T_s - \frac{1}{2}\left[(T_j)_{1'} + (T_f)_{1'}\right] .$$

Since

$$\Delta T_1 = (T_f)_{1'} - (T_j)_{1'} ,$$

the above equation can be rearranged to give

$$\Delta t_0 = T_s - (T_f)_{1'} + \frac{1}{2}\Delta T_1$$

$$= T_s - (T_f)_0 + \frac{1}{2}\,\frac{q_s/F}{C_p'(1 + r_1)} \qquad (33)$$

where

$$(T_f)_0 = (T_f)_{1'} .$$

The maximum value of $(T_f)_0$ must generally be limited in order to control scale formation. In this study, the values of T_s and $(T_f)_0$ are assumed to be fixed.

B). Δt in the n-th Effect, Δt_n .

Referring to a stage in section HR-n of Figure 3, it can be seen that the effective Δt for heat transfer in the infinite stage operation is given by

$$\frac{uw + xy}{2} = uw = xy .$$

For N stage operation, the effective Δt becomes

$$\frac{vw + xy}{2}$$

because the condensation temperature is constant within each stage. Therefore, the loss in the effective Δt for heat transfer is

$$\frac{uw + xy}{2} - \frac{vw + xy}{2} = \frac{uw - vw}{2} .$$

In other words, the loss in Δt for heat transfer is one-half of the temperature drop from stage to stage. Since the average temperature drop from stage to stage in the n-th effect is given by

$$\frac{(T_f)_{n-1} - (T_f)_n}{N_n} . \qquad (34)$$

where N_n is the number of stages in the n-th effect, the average loss in effective Δt_n for heat transfer is

$$\Delta t_{n,\ loss} = \frac{(T_f)_{n-1} - (T_f)_n}{2N_n} . \qquad (35)$$

Therefore, the effective Δt_n is given by

$$\Delta t_n = \Delta T_n - \alpha_n - \Delta t_{n,\ loss}$$

$$= \frac{\frac{q_s}{F}\ \frac{1}{C_p} + \left(1 - \frac{C_F}{(C_f)_{n-1}}\right)_n}{1 + \frac{r_n C_F}{(C_f)_{n-1}}} - \alpha_n - \frac{(T_f)_{n-1} - (T_f)_n}{2N_n} ,$$

$$n = 1,\ 2,\ 3. \qquad (36)$$

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$$n = 1, 2, 3. \tag{36}$$

3-11. Heat Transfer Area Requirements in the Brine Heater and the n-th Effect.

Equations for the heat loads and temperature difference for heat transfer in the brine heater Δt_0 and that in the n-th effect, Δt_n, have been developed in sections 3-4 and 3-10, respectively. These are used to calculate the heat transfer areas by the equation

$$A = \frac{q}{U\ (\Delta t)} \qquad (37)$$

By assuming U is constant in the brine heater and each effect, and substituting equation (33) into equation (37), the heat transfer area in the brine heater A_0, can be obtained as

$$A_0 = \frac{q_s}{U\left\{T_s - (T_f)_0 + \frac{1}{2}\,\frac{q_s/F}{C_p(1+r_1)}\right\}} \qquad (38)$$

Substituting equations (11) and (36) into equation (37) yields the heat transfer area in the n-th effect, A_n, as

$$A_n = \frac{F\left[\frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n}\right]\lambda}{U\left\{\frac{\frac{q_s}{F}\ \frac{1}{C_p} + \left(1 - \frac{C_F}{(C_f)_{n-1}}\right) n}{1 + \frac{r_n C_F}{(C_f)_{n-1}}} - \alpha_n - \frac{(T_f)_{n-1} - (T_f)_n}{2N_n}\right\}}$$

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$$n = 1, 2, 3. \tag{39}$$

3-12. Pumping Head for the Recirculation Pump, J_n.

The recirculation pump, J_n, takes in recycle brine R_n of concentration $(C_f)_n$ at temperature $(T_f)_n$, pressurizes it to a pressure sufficiently high so that the recycle brine does not boil within the heating tubes. The highest temperature to which this brine stream is heated is $(T_f)_{n-1}$. Thus, the pumping head $(\Delta P_n)/\rho$ required for J_n can be evaluated as,

$$\frac{(\Delta P)_n}{\rho} = \frac{1}{\rho} (1 + \eta_f) \left[(\overline{P})_{n-1} - (\overline{P})_n \right] , \qquad (40)$$

where η_f is the fractional excess pumping head required due to friction losses and $(\overline{P})_n$ is the vapor pressure of the brine at concentration $(C_f)_n$ and temperature $(T_f)_n$.

The vapor pressure of brine at a given temperature is less than the vapor pressure of pure water at the same temperature due to the vapor pressure depression of the solution. Thus,

$$(\overline{P})_n = (P^o)_n - (\beta)_n ,$$

and

$$(\overline{P})_{n-1} = (P^o)_{n-1} - (\beta)_{n-1} ,$$

where $(P^o)_n$ and $(P^o)_{n-1}$ are the vapor pressures of pure water at temperatures $(T_f)_n$ and $(T_f)_{n-1}$ respectively and $(\beta)_n$ and $(\beta)_{n-1}$ are vapor pressure depressions of the brine streams at temperatures $(T_f)_n$ and $(T_f)_{n-1}$ and concentrations $(C_f)_n$ and $(C_f)_{n-1}$, respectively.

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If we assume that

$$(\beta)_n = (\beta)_{n-1} ,$$

we obtain

$$(\overline{P})_{n-1} - (\overline{P})_n = (P^o)_{n-1} - (P^o)_n ,$$

and equation (40) becomes

$$\frac{(\Delta P)_n}{\rho} = \frac{1}{\rho} (1 + \eta_f) \left[(P^o)_{n-1} - (P^o)_n \right]$$

Within the operating temperature range of the MEMS system, the vapor pressure of water may be represented by

$$\ln P^o = - \frac{\lambda}{RT} + D$$

where λ is the latent heat of vaporization and D is an integration constant. On rearranging, we obtain

$$P^o = e^D e^{-\frac{\lambda}{RT}} = B' e^{-\frac{\lambda}{RT}}$$

where $B' = e^D$. By assuming a constant value of 1000 Btu/lb_m for λ and from the steam table at two temperatures, B' is evaluated to have a value of 1.523×10^9 lb_{f/ft^2} .

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Substituting equation (42) into equation (41) gives

$$\frac{\Delta P_n}{\rho} = \frac{B'}{\rho}(1+\eta_f)\left\{\exp\left(-\frac{\lambda}{(T_f)_{n-1}}\right) - \exp\left(-\frac{\lambda}{(T_f)_n}\right)\right\},$$

$$n = 1, 2, 3. \qquad (43)$$

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CHAPTER 4

ECONOMIC ANALYSIS

The water production cost depends primarily on the capital cost and operating cost of the plant. Only those costs which are most significantly affected by changes in the design variables are considered in this study. The cost of the plant site, labor cost, overhead costs and insurance costs are not considered here as they are little affected by changes in the design variables.

The capital cost consists of three items:

(a). The heat transfer area cost,

(b). The recirculation pump cost,

(c). The outer shell cost.

The operating cost consists of four items:

(a). Feed brine cost,

(b). Cooling water cost,

(c). Steam cost,

(d). Power cost for recirculation pumping in each effect.

Each cost item is expressed as the cost per 1000 gallons of fresh water produced in the whole plant. The following notation is used to represent the various cost items:

E_1 = Steam cost,

E_2 = Capital cost of brine heater,

E_3^n = Capital cost of the heat transfer area in the n-th effect,

E_4^n = Power cost for recirculation pumping in the n-th effect,

E_5^n = Capital cost for recirculation pump in the n-th effect,

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4-1. The Capital Costs.

The capital costs of the principal items of equipment are evaluated per unit of production in a unit time. The annual capitalization charges for these equipment items are calculated at 0.074 of the initial cost per year, as recommended in the Office of Saline Water procedures (10). A load factor of 330 on-stream days per year will be assumed. Therefore, the capitalization charge, ψ, is 9.4×10^{-6} of the initial cost per hour on-stream.

(a) Brine Heater Cost, E_2

The brine heater cost is assumed to be proportional to the brine heater area, A_0, which is given by equation (38). Therefore, the capital cost per 1000 gallons of water production per hour, E_2, is given by

$$E_2 = \frac{\psi C_B A_0 W}{W_n}$$

$$= C_{ht} \frac{q_s}{F} \frac{1}{U\left[a - (T_f)_0 + \frac{1}{2} \frac{q_s/F}{c_p(1 + r_1)}\right]} \frac{W}{1 - \frac{C_F}{(C_f)_3}} \tag{44}$$

where

C_B = the capital cost per unit of heat transfer area for the brine heater,

C_{ht} = ψC_B ,

W = mass equivalent to 1000 gallons of water,

a = T_s , i.e., the steam temperature.

This cost includes both the heat transfer area and the outer shell costs.

E_7 = Feed brine cost,

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This cost includes both the heat transfer area and the outer shell costs.

(b) Heat Transfer Area Cost in the n th Effect, E_3^n.

The heat transfer area for the n-th effect, A_n, is given by equation (39). Therefore, the equation for the capital cost per 1000 gallons of water produced can be represented by

$$E_3^n = \frac{\psi C_H A_n W}{\Sigma W_n}$$

$$= C_{cd} \frac{\left[\frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n}\right]\lambda}{U\left\{\frac{\frac{q_s}{F}\frac{1}{C_p} + \alpha_n\left(1 - \frac{C_F}{(C_f)_{n-1}}\right)}{1 + \frac{r_n C_f}{(C_f)_{n-1}}} - \alpha_n - \frac{(T_f)_{n-1} - (T_f)_n}{2N_n}\right\}} \cdot \frac{W}{1 - \frac{C_F}{(C_f)_3}}, \qquad n = 1, 2, 3, \tag{45}$$

where

C_H = the capital cost per unit of heat transfer area,

$C_{cd} = \psi C_H$.

(c) Recirculation Pump Cost in the n-th Effect, E_5^n .

The pump cost is assumed to be proportional to its power rate. Since the pumping head in the n-th effect is given by equation (43), the cost equation for the pump J_n can be written as

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$$E_5^n$$

$$= \psi C_J \quad \frac{\Delta P_n}{\rho} \quad R_n \quad \frac{W}{\Sigma W_n}$$

$$= \psi C_J \quad \frac{B\,(1 + \eta_f) \quad r_n C_F W}{(1 - \frac{C_F}{(C_f)_3})\ (C_f)_{n-1}} \left\{ \exp\left(- \frac{\lambda}{R(T_f)_{n-1}} \right) - \exp\left(- \frac{\lambda}{R(T_f)_n} \right) \right\}$$

$$n = 1, 2, 3. \qquad (46)$$

(d) Outer Shell Cost, E_6.

Because of lack of information, E_6 will be considered to be a constant value.

4-2. The Operating Costs.

(a) Steam Cost, E_1.

The amount of steam used in the brine heater is q_s/λ_s , and the steam cost per 1000 gallons of water produced is given by

$$E_1 = C_{st} \quad \frac{q_s/\lambda_s}{\Sigma W_n}$$

$$= C_{st} \quad \frac{q_s}{F} \quad \frac{1}{\lambda_s} \quad \frac{W}{1 - \frac{C_F}{(C_f)_3}} \qquad (47)$$

where C_{st} is the unit steam cost.

(b) Feed Brine Cost, E_7.

The cost of the brine feed to the system is proportional to the quantity of the feed which is given by

$$
\begin{aligned}
E_5^n &= \psi\, C_J \; \frac{\Delta P_n}{\rho} \; R_n \; \frac{W}{\Sigma W_n} \\
&= \psi\, C_J \; \frac{B\,(1+\eta_f)\quad r_n C_F W}{(1-\frac{C_F}{(C_f)_3})\;(C_f)_{n-1}} \left\{ \exp\left(-\frac{\lambda}{R(T_f)_{n-1}}\right) - \exp\left(-\frac{\lambda}{R(T_f)_n}\right) \right\}
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The cost of the brine feed to the system is proportional to the quantity of the feed which is given by

$$F = \frac{\Sigma W_n}{1 - \frac{C_F}{(C_f)_3}}$$

Therefore, the cost per 1000 gallons of water produced is

$$E_7 = p_c \frac{F}{\Sigma W_n} W$$

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where p_c is the unit feed water cost.

(c) Cooling Water Cost, E_8 .

Cooling water cost is proportional to the amount of the cooling water, R_4, which is given by equation (20). Therefore, we have the following equation for E_8 .

$$E_8 = c_c \left\{ \frac{\frac{q_s}{F} \frac{1}{C_p} \frac{1}{1 - \frac{C_F}{(C_f)_3}} + \alpha_3}{(T_f)_3 - (T_j)_3} - \frac{1}{1 - \frac{C_F}{(C_f)_3}} \right\} W \tag{49}$$

where c_c is the unit cooling water cost.

(d) The Power Cost for the Recycle Pump in the n-th Effect, E_n^n.

The pumping head for the recycle pump J_n , and the amount of recycle brine are given in equations (43) and (5) respectively.

$$F = \frac{\Sigma W_n}{1 - \dfrac{C_F}{(C_f)_3}}$$

Therefore, the cost per 1000 gallons of water produced is

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(d) The Power Cost for the Recycle Pump in the n-th Effect, E_n^n.

The pumping head for the recycle pump J_n , and the amount of recycle brine are given in equations (43) and (5) respectively.

The energy cost per 1000 gallons of water produced associated with the operation of the recycle pump in the n-th effect can be calculated by the following equation:

$$E_4^n = C_e \frac{W}{\Sigma W_n} \frac{1}{\eta_p} R_n \frac{(\Delta P)_n}{\rho}$$

where C_e is the unit cost of power and η_p is the pumping efficiency. By substituting equations (5), (8), and (43) into the above equation, we obtain

$$E_4^n = C_3 \frac{B(1+\eta_f)}{\eta_p} r_n \frac{C_F}{(C_f)_{n-1}} \frac{1}{1 - \frac{C_F}{(C_f)_3}} \left\{ \exp\left(-\frac{\lambda}{R(C_f)_{n-1}}\right) - \exp\left(-\frac{\lambda}{R(C_f)_n}\right) \right\}$$

$$n = 1, 2, 3. \qquad (50)$$

4-3. The Water Production Cost, S

The water cost per 1000 gallons of fresh water production is the sum of the various cost items we have described, that is

$$S = E_1 + E_2 + \sum_{n=1}^{3} E_3^n + \sum_{n=1}^{3} E_4^n + \sum_{n=1}^{3} E_5^n + E_6 + E_7 + E_8$$

$$= C_{st} \frac{q_s}{F} \frac{1}{\lambda_s} \frac{1}{1 - \frac{C_F}{(C_f)_3}} + C_{ht} \frac{q_s}{F} \frac{1}{U\left\{a - (T_f)_0 + \frac{1}{2} \frac{q_s/F}{C_p(1+r_1)}\right\}} \frac{W}{1 - \frac{C_F}{(C_f)_3}}$$

$$+ E_6 + p_c \frac{W}{1 - \frac{C_F}{(C_f)_3}}$$

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$$= C_{st} \frac{q_s}{F} \frac{1}{\lambda_s} \frac{1}{1-\dfrac{c_F}{(c_f)_3}} + C_{ht} \frac{q_s}{F} \frac{1}{U\left\{a-(T_f)_0+\dfrac{1}{2}\dfrac{q_s/F}{c_p(1+r_1)}\right\}} \frac{W}{1-\dfrac{c_F}{(c_f)_3}}$$

$$+ E_6 + p_c \frac{W}{1-\dfrac{c_F}{(c_f)_3}}$$

$$+ \sum_{n=1}^{3} C_{cd} \frac{\left\{ \frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n} \right\} \lambda}{U \left\{ \left[\frac{\frac{q_s}{F} \frac{1}{C_p} + \alpha_n \left(1 - \frac{C_F}{(C_f)_{n-1}}\right)}{1 + \frac{r_n C_f}{(C_f)_{n-1}}} \right] - \alpha_n - \frac{(T_f)_{n-1} - (T_f)_n}{2N_n} \right\}} \frac{W}{1 - \frac{C_F}{(C_f)_3}}$$

$$+ \sum_{n=1}^{3} C_{pp} \frac{C_F}{(C_f)_{n-1}} r_n \frac{B}{\rho} \left\{ \exp\left(- \frac{\lambda}{R(T_f)_{n-1}}\right) - \exp\left(- \frac{\lambda}{R(T_f)_n}\right) \right\} \frac{W}{1 - \frac{C_F}{(C_f)_3}}$$

$$+ cc \left\{ \frac{\frac{q_s}{F} \frac{1}{C_p} \frac{1}{1 - \frac{C_F}{(C_f)_3}} + \alpha_3}{(T_f)_3 - (T_j)_3} - \frac{1}{1 - \frac{C_F}{(C_f)_3}} \right\} W \qquad (51)$$

where

$$C_{pp} = \frac{Ce}{n_p} + C_J$$

$$B = (1 + n_f) B'$$

$$+ \sum_{n=1}^{3} C_{cd} \frac{\left\{ \frac{C_F}{(C_f)_{n-1}} - \frac{C_F}{(C_f)_n} \right\} \lambda}{U \left\{ \left[\frac{\frac{q_s}{F} \frac{1}{C_p} + \alpha_n \left(1 - \frac{C_F}{(C_f)_{n-1}}\right)}{1 + \frac{r_n C_f}{(C_f)_{n-1}}} \right] - \alpha_n - \frac{(T_f)_{n-1} - (T_f)_n}{2N_n} \right\}} \frac{W}{1 - \frac{C_F}{(C_f)_3}}$$

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CHAPTER 5.

OPTIMIZATION

The performance equations of the MEMS process are described in chapter 3 and the cost equations of the water production are derived in chapter 4. Armed with these equations we can proceed ~~by using the optimization technique~~ to determine the optimal conditions of the process. Since a discrete form of the maximum principle is effective for seeking the optimal conditions of a sequential multistage multidecision process, we shall use it in conjuction with search techniques to optimize the process (11). Two search techniques are used here: one is the parametric search and the other is the simplex method. The results of the numerical solution and the comparision of the two approaches are given in sections 5-3, 5-4, and 5-5. The computer programs for the two methods are presented in the Appendix.

5-1. The Performance Equations

From equation (51) it is known that the water cost S is a function of thirteen variables: q_s/F, C_F, $(C_f)_1$, $(C_f)_2$, $(C_f)_3$, r_1, r_2, r_3, $(T_f)_0$, $(T_f)_1$, $(T_f)_2$, $(T_f)_3$, and $(T_j)_3$. But C_F, the brine concentration of sea water feed, is fixed and assumed to be 3.5% (5), and $(T_f)_0$, the temperature of the brine stream leaving the brine heater, is fixed because of the need for the controling of the scale formation. The sea water temperature $(T_j)_3$ is rarely changed and is assumed constant. Therefore we have

$$S = S(q_s/_F, (C_f)_1, (C_f)_2, (C_f)_3, (T_f)_1, (T_f)_2, (T_f)_3, r_1, r_2, r_3). \qquad (52)$$

However, these ten variables are not all independent; equation (15) gives three relations between these variables. We have then seven independent variables. According to the maximum principle we classified the variables into

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However, these ten variables are not all independent; equation (15) gives three relations between these variables. We have then seven independent variables. According to the maximum principle we classified the variables into

state variables x and decision variables θ. If we define the concentration $(C_f)_n$ and the water cost S as state variables, then the recycle ratio r_n and the brine temperature $(T_f)_n$ are the decision variables. Therefore we may write

$$x_1^n = (C_f)_n \qquad n = 0, 1, 2, 3. \tag{53}$$

where

$$x_1^0 = C_F,$$

$$\theta_1^n = r_n, \qquad n = 1, 2, 3, \tag{54}$$

and

$$\theta_2^n = (T_f)_n \qquad n = 0, 1, 2, 3. \tag{55}$$

Equation (15) then becomes

$$\ln x_1^n = \ln \frac{x_1^{n-1} + \theta_1^n x_1^n}{1 + \theta_1^n} + \frac{C_p}{\lambda}(\theta_2^{n-1} - \theta_2^n), \tag{56}$$

$$n = 1, 2, 3.$$

As this equation includes the previous decision θ_2^{n-1}, that is, it has memory in decision, we introduce a new decision variable θ_3^n and a new state variable x_3^n such that

$$\theta_3^n = \theta_2^n - \theta_2^{n-1}, \qquad n = 1, 2, 3, \tag{57}$$

and

$$x_3^n = \theta_2^n, \qquad n = 1, 2, 3, \tag{58}$$

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$$\ln x_1^n = \ln \frac{x_1^{n-1} + \theta_1^n x_1^n}{1 + \theta_n^n} - \frac{C_p}{\lambda} \theta_3^n, \qquad n = 1, 2, 3. \tag{60}$$

From equation (51) the state variable x_2^n for water cost is defined as follows:

$$x_2^0 = C_{st} \frac{q_s}{F} \frac{1}{\lambda_s} \frac{W}{1 - \frac{C_F}{x_1^3}} + E_6$$

$$+ C_{ht} \frac{q_s}{F} \frac{1}{U(a - x_3^0 + \frac{q_s/F}{2C_p(1+ \theta_1^1)})} \frac{W}{1 - \frac{C_F}{x_1^3}}, \tag{61}$$

$$x_2^n = x_2^{n-1} + C_{cd} \frac{\left[\frac{C_F}{x_1^{n-1}} - \frac{C_F}{x_1^n}\right] \lambda}{\frac{\frac{q}{F}\frac{1}{C_p} + \alpha_n \left(1 - \frac{C_F}{x_1^{n-1}}\right)}{1 + \theta_1^n \frac{C_F}{x_1^{n-1}}} - \alpha_n + \frac{\theta_3^n}{2X^n}} \cdot \frac{W}{1 - \frac{C_F}{x_1^3}}$$

$$+ C_{pp} \frac{C_F}{x_1^{n-1}} \theta_1^n \frac{B}{\rho} \{\exp \left[- \frac{\lambda}{Rx_3^{n-1}}\right] - \exp \left[\frac{-\lambda}{R(x_3^{n-1} + \theta^n)}\right] \frac{W}{1 - \frac{C_F}{x_1^3}}, \tag{62}$$

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$$\left\{x_2^3\right\} = x_2^3 + (p_c - c_c)\,\frac{W}{1 - \dfrac{c_F}{x_1^3}} + c_c\,\frac{\dfrac{q_s}{F}\cdot\dfrac{W}{1 - \dfrac{c_F}{x_1^3}} + \alpha_3 W}{(x_3^2 + \theta_3^3 - 545)}\,, \tag{63}$$

where

$$\alpha_1 = 1.01 + \frac{1}{0.03}\cdot\frac{\dfrac{c_F + \theta_1^1 x_1^1}{1 + \theta_1^1} + x_1^1}{2} \tag{64}$$

$$\alpha_2 = 1.0075 + \frac{1}{0.0347}\cdot\frac{\dfrac{x_1^1 + \theta_1^2 x_1^2}{1 + \theta_1^2} + x_1^2}{2} \tag{65}$$

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The sea water temperature is assumed constant and equal to 85°F or 545°R. We must note that x_1^3 appears in nearly every equation. The same is true for q_s/F. Therefore the values of x_1^3 and q_s/F must be given in advance before we proceed to optimize the cost function. The optimization problem we have imposed is as follows:

Find a sequence of decisions $\theta_1^1, \theta_1^2, \theta_1^3, \theta_2^1, \theta_3^2, \theta_3^3$ to minimize x_2^3 with x_1^3 and q_s/F preassigned.

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Once the values of x_1^3 and q_s/F are known, the optimal value of θ is sought by the algorithm of the maximum principle, but the optimal values of x_1^3 and q_s/F must be found by one of the two search techniques mentioned before.

5-2. Search for Optimum by the Maximum Principle

(a) Differentiation of State Variables.

The differentiations of the state variables with respect to the decision variables and state variables are given below. These are used to determine the adjoint variable and the derivatives of the Hamiltonian funcitons in the following sections.

(1). x_1^n

$$\frac{\partial x_1^n}{\partial \theta_1^n} = \frac{x_1^n(x_1^n - x_1^{n-1})}{x_1^{n-1}(1 + \theta_1^n)} , \qquad n = 1, 2, 3 \qquad (67)$$

$$\frac{\partial x_1^n}{\partial \theta_3^n} = -\frac{C_p x_1^n(x_1^{n-1} + \theta_1^n x_1^n)}{\lambda x_1^{n-1}} , \qquad n = 1, 2, 3 \qquad (68)$$

$$\frac{\partial x_1^n}{\partial x_1^{n-1}} = \frac{x_1^n}{x_1^{n-1}} , \qquad n = 2, 3, \qquad (69)$$

$$\frac{\partial x_1^n}{\partial x_2^{n-1}} = 0 , \qquad n = 2, 3, \qquad (70)$$

$$\frac{\partial x_1^n}{\partial x_3^{n-1}} = 0 , \qquad n = 2, 3. \qquad (71)$$

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(2). x_2^n

$$\frac{\partial x_2^n}{\partial \theta_1^n} = C_{cd}\,\frac{\lambda}{U}\,\frac{W}{1-\frac{C_F}{x_1^3}}\left\{\frac{C_F\,\frac{\partial x_1^n}{\partial \theta_1^n}}{(x_1^n)^2\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N_n}\right]} - \frac{\left[\frac{C_F}{x_1^{n-1}} - \frac{C_F}{x_1^n}\right]\frac{\partial \Delta T^n}{\partial \theta_1^n}}{\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N_n}\right]^2}\right\}$$

$$+ C_{pp}\,\frac{C_F}{x_1^{n-1}}\cdot\frac{B}{\rho}\;\left\{\exp\left(-\frac{\lambda}{Rx_3^{n-1}}\right) - \exp\left[\frac{-\lambda}{R(x_3^{n-1}+\theta_3^n)}\right]\right\}\;\frac{W}{1-\frac{C_F}{x_1^3}}\,, \qquad (72)$$

$$n = 1,\ 2,\ 3,$$

where

$$\Delta T^n = \frac{\frac{q}{F}\cdot\frac{1}{C_p} + \alpha_n\left[1-\frac{C_F}{x_1^{n-1}}\right]}{1+\theta_1^n\,\frac{C_F}{x_1^{n-1}}}\,, \qquad (73)$$

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$$n = 1,\ 2,\ 3,$$

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$$- C_{pp}\,\frac{C_F}{x_1^2}\,\theta_1^3\,\frac{B}{\rho}\cdot\frac{WC_F\,\frac{\partial x_1^3}{\partial \theta_1^3}}{\left[x_1^3 - C_F\right]^2}\left\{\exp\left[-\frac{\lambda}{Rx_3^2}\right] - \exp\left[\frac{-\lambda}{R(x_3^2+\theta_3^3)}\right]\right\}, \qquad (77)$$

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$$\left\{\frac{\partial x_2^3}{\partial \theta_1^3}\right\} = \frac{\partial x_2^3}{\partial \theta_1^3} - C_{cd}\,\frac{1}{U}\cdot\frac{W.C_F\cdot\dfrac{\partial x_1^3}{\partial \theta_1^3}}{\left[x_1^3-C_F\right]^2}\cdot\frac{\left[\dfrac{C_F}{x_1^2}-\dfrac{C_F}{x_1^3}\right]\cdot\lambda}{\left[\Delta T^3-\alpha^3+\dfrac{\theta_3^3}{2N_3}\right]}$$

$$-\,C_{pp}\,\frac{C_F}{x_1^2}\,\theta_1^3\,\frac{B}{\rho}\cdot\frac{WC_F\,\dfrac{\partial x_1^3}{\partial \theta_1^3}}{\left[x_1^3-C_F\right]^2}\left\{\exp\left[-\frac{\lambda}{Rx_3^2}\right]-\exp\left[\frac{-\lambda}{R(x_3^2+\theta_3^3)}\right]\right\}, \qquad (77)$$

$$\left\{ \frac{\partial x_2^3}{\partial \theta_3^3} \right\} = \frac{\partial x_2^3}{\partial \theta_3^3} - C_{cd} \frac{1}{U} \frac{WC_F \frac{\partial x_1^3}{\partial \theta_3^3}}{\left[x_1^3 - C_F\right]^2} \cdot \frac{\left[\frac{C_F}{x_1^2} - \frac{C_F}{x_1^3}\right] \lambda}{\left[\Delta T^3 - \alpha_3 + \frac{\theta_3^3}{2N_3}\right]}$$

$$- C_{pp} \frac{C_F}{x_1^2} \theta_1^3 \frac{B}{\rho} \cdot \frac{WC_F \frac{\partial x_1^3}{\partial \theta_3^3}}{\left[x_1^3 - C_F\right]^2} \left\{ \exp\left[- \frac{\lambda}{Rx_3^2}\right] - \exp\left[\frac{-\lambda}{R(x_3^2 + \theta_3^3)}\right] \right\}, \qquad (78)$$

$$\frac{\partial x_2^3}{\partial x_1^{n-1}} = C_{cd} \frac{\lambda}{U} \cdot \frac{W}{1 - \frac{C_F}{x_1^3}} C_F \left\{ \frac{- \frac{1}{(x_1^{n-1})^2} + \frac{\frac{\partial x_1^n}{\partial x_1^{n-1}}}{(x_1^n)^2}}{\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N^n}\right]} - \frac{\left[\frac{1}{x_1^{n-1}} - \frac{1}{x_1^n}\right] \frac{\partial \Delta T^n}{\partial x_1^{n-1}}}{\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N_n}\right]^2} \right\}$$

$$-C_{pp} \frac{C_F}{(x_1^{n-1})^2} \theta_1^n \frac{B}{\rho} \left\{ \exp\left[- \frac{\lambda}{Rx_3^{n-1}}\right] - \exp\left[\frac{-\lambda}{R(x_3^{n-1} + \theta_3^n)}\right] \right\} \frac{W}{1 - \frac{C_F}{x_1^3}}, \qquad (79)$$

$$n = 2,\ 3,$$

$$\left\{ \frac{\partial x_2^3}{\partial \theta_3^3} \right\} = \frac{\partial x_2^3}{\partial \theta_3^3} - C_{cd} \frac{1}{U} \frac{WC_F \frac{\partial x_1^3}{\partial \theta_3^3}}{\left[x_1^3 - C_F\right]^2} \cdot \frac{\left[\frac{C_F}{x_1^2} - \frac{C_F}{x_1^3}\right] \lambda}{\left[\Delta T^3 - \alpha_3 + \frac{\theta_3^3}{2N_3}\right]}$$

$$- C_{pp} \frac{C_F}{x_1^2} \theta_1^3 \frac{B}{\rho} \cdot \frac{WC_F \frac{\partial x_1^3}{\partial \theta_3^3}}{\left[x_1^3 - C_F\right]^2} \left\{ \exp\left[-\frac{\lambda}{Rx_3^2}\right] - \exp\left[\frac{-\lambda}{R(x_3^2 + \theta_3^3)}\right] \right\}, \qquad (78)$$

$$\frac{\partial x_2^3}{\partial x_1^{n-1}} = C_{cd} \frac{\lambda}{U} \cdot \frac{W}{1 - \frac{C_F}{x_1^3}} C_F \left\{ \frac{-\frac{1}{(x_1^{n-1})^2} + \frac{\frac{\partial x_1^n}{\partial x_1^{n-1}}}{(x_1^n)^2}}{\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N^n}\right]} - \frac{\left[\frac{1}{x_1^{n-1}} - \frac{1}{x_1^n}\right] \frac{\partial \Delta T^n}{\partial x_1^{n-1}}}{\left[\Delta T^n - \alpha_n + \frac{\theta_3^n}{2N_n}\right]^2} \right\}$$

$$-C_{pp} \frac{C_F}{(x_1^{n-1})^2} \theta_1^n \frac{B}{\rho} \left\{ \exp\left[-\frac{\lambda}{Rx_3^{n-1}}\right] - \exp\left[\frac{-\lambda}{R(x_3^{n-1} + \theta_3^n)}\right] \right\} \frac{W}{1 - \frac{C_F}{x_1^3}}, \qquad (79)$$

$n = 2, 3,$

$$\left\{\frac{\partial x_2^3}{\partial x_1^2}\right\} = \frac{\partial x_2^3}{\partial x_1^2} - C_{cd}\,\frac{1}{U}\cdot\frac{WC_F\,\dfrac{\partial x_1^3}{\partial x_1^2}}{(x_1^3 - C_F)^2}\cdot\frac{\left[\dfrac{C_F}{x_1^2} - \dfrac{C_F}{x_1^3}\right]\lambda}{\left[\Delta T^3 - \alpha_n + \dfrac{\theta_3^3}{2N_3}\right]}$$

$$-C_{pp}\,\frac{C_F}{x_1^2}\,\theta_1^3\,\frac{B}{\rho}\,\frac{WC_F\,\dfrac{\partial x_1^3}{\partial x_1^2}}{(x_1^3 - C_F)^2}\left\{\exp\left(-\frac{\lambda}{Rx_3^2}\right) - \exp\left(\frac{-\lambda}{R(x_3^2 + \theta_3^3)}\right)\right\}, \qquad (80)$$

where

$$\frac{\partial\,\Delta T^n}{\partial x_1^{n-1}} = \frac{C_F(\alpha_n + \Delta T^n\,\theta_1^n)}{x_1^{n-1}(x_1^{n-1} + \theta_1^n\,C_F)}\,. \qquad n = 2,\ 3, \qquad (81)$$

$$\frac{\partial x_2^n}{\partial x_2^{n-1}} = 1, \qquad n = 2,\ 3, \qquad (82)$$

$$\frac{\partial x_2^n}{\partial x_3^{n-1}} = C_{pp}\,\frac{C_F}{x_1^{n-1}}\,\theta_1^n\,\frac{B}{\rho}\,\frac{W}{1 - \dfrac{C_F}{x_1^3}}\cdot\frac{\lambda}{R}\left\{\frac{1}{(x_3^{n-1})^2}\,\exp\left(-\frac{\lambda}{Rx_3^{n-1}}\right)\right.$$

$$\left. - \frac{1}{(x_3^{n-1} + \theta_3^n)^2}\,\exp\left(\frac{-\lambda}{R(x_3^{n-1} + \theta_3^n)}\right)\right\}, \qquad (83)$$

$$n = 2,3.$$

$$\left\{\frac{\partial x_2^3}{\partial x_1^2}\right\} = \frac{\partial x_2^3}{\partial x_1^2} - C_{cd}\,\frac{1}{U}\cdot\frac{WC_F\,\dfrac{\partial x_1^3}{\partial x_1^2}}{(x_1^3 - C_F)^2}\cdot\frac{\left[\dfrac{C_F}{x_1^2} - \dfrac{C_F}{x_1^3}\right]\lambda}{\left[\Delta T^3 - \alpha_n + \dfrac{\theta_3^3}{2N_3}\right]}$$

$$-C_{pp}\,\frac{C_F}{x_1^2}\,\theta_1^3\,\frac{B}{\rho}\,\frac{WC_F\,\dfrac{\partial x_1^3}{\partial x_1^2}}{(x_1^3 - C_F)^2}\left\{\exp\left(-\frac{\lambda}{Rx_3^2}\right) - \exp\left(\frac{-\lambda}{R(x_3^2 + \theta_3^3)}\right)\right\}, \qquad (80)$$

where

$$\frac{\partial\,\Delta T^n}{\partial x_1^{n-1}} = \frac{C_F(\alpha_n + \Delta T^n\,\theta_1^n)}{x_1^{n-1}(x_1^{n-1} + \theta_1^n\,C_F)}\;. \qquad n = 2,\ 3, \qquad (81)$$

$$\frac{\partial x_2^n}{\partial x_2^{n-1}} = 1, \qquad n = 2,\ 3, \qquad (82)$$

$$\frac{\partial x_2^n}{\partial x_3^{n-1}} = C_{pp}\,\frac{C_F}{x_1^{n-1}}\,\theta_1^n\,\frac{B}{\rho}\,\frac{W}{1 - \dfrac{C_F}{x_1^3}}\cdot\frac{\lambda}{R}\left\{\frac{1}{(x_3^{n-1})^2}\,\exp\left(-\frac{\lambda}{Rx_3^{n-1}}\right)\right.$$

$$\left. - \frac{1}{(x_3^{n-1} + \theta_3^n)^2}\,\exp\left(\frac{-\lambda}{R(x_3^{n-1} + \theta_3^n)}\right)\right\}, \qquad (83)$$

$$n = 2,3.$$

(3). x_3^n

$$\frac{\partial x_3^n}{\partial \theta_1^n} = 0, \qquad n = 1, 2, 3, \tag{84}$$

$$\frac{\partial x_3^n}{\partial \theta_3^n} = 1, \qquad n = 1, 2, 3, \tag{85}$$

$$\frac{\partial x_3^n}{\partial x_1^{n-1}} = 0, \qquad n = 2, 3, \tag{86}$$

$$\frac{\partial x_3^n}{\partial x_2^{n-1}} = 0, \qquad n = 2, 3, \tag{87}$$

$$\frac{\partial x_3^n}{\partial x_3^{n-1}} = 1, \qquad n = 2, 3. \tag{88}$$

(b) Adjoint Variables z_i^N

Since

$$c_1 = 0, \qquad c_2 = 1, \qquad c_3 = 0,$$

we can write

$$z_1^3 = 0, \qquad z_2^3 = 1, \qquad z_3^3 = 0,$$

However, since x_1^3 is prefixed,

$$z_1^3 \neq c_1 .$$

Then H^3 becomes

(3). x_3^n

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$$\frac{\partial x_3^n}{\partial \theta_3^n} = 1, \qquad n = 1, 2, 3, \tag{85}$$

$$\frac{\partial x_3^n}{\partial x_1^{n-1}} = 0, \qquad n = 2, 3, \tag{86}$$

$$\frac{\partial x_3^n}{\partial x_2^{n-1}} = 0, \qquad n = 2, 3, \tag{87}$$

$$\frac{\partial x_3^n}{\partial x_3^{n-1}} = 1, \qquad n = 2, 3. \tag{88}$$

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However, since x_1^3 is prefixed,

$$z_1^3 \neq c_1 .$$

Then H^3 becomes

$$H^3 = z_1^3 x_1^3 + x_2^3 . \qquad (89)$$

Differentiating H^3 with respect to θ_3^3 yields

$$\frac{\partial H^3}{\partial \theta_3^3} = z_1^3 \frac{\partial x_1^3}{\partial \theta_3^3} + \frac{\partial x_2^3}{\partial \theta_3^3} . \qquad (90)$$

Setting $\frac{\partial H^3}{\partial \theta_3^3} = 0$ yields

$$z_1^3 = \frac{-\dfrac{\partial x_2^3}{\partial \theta_3^3}}{\dfrac{\partial x_1^3}{\partial \theta_3^3}} . \qquad (91)$$

(c) Adjoint Variables z_i^n

z_i^N derived in the last section is used to calculate z_i^n in the following equations. In the actual calculation the values of the differentiation of the state variables in section (a) are substituted into the equation of z_i^n.

$$z_1^2 = z_1^3 \frac{\partial x_1^3}{\partial x_1^2} + z_2^3 \frac{\partial x_2^3}{\partial x_1^2} = z_1^3 \frac{\partial x_1^3}{\partial x_1^2} + \frac{\partial x_2^3}{\partial x_1^2} , \qquad (92)$$

$$z_2^2 = z_1^3 \frac{\partial x_1^3}{\partial x_2^2} + z_2^3 \frac{\partial x_2^3}{\partial x_2^2}$$

$$= 1, \qquad (93)$$

$$z_3^2 = z_1^3 . \frac{\partial x_1^3}{\partial x_3^2} + z_2^3 \frac{\partial x_2^3}{\partial x_3^2} = \frac{\partial x_2^3}{\partial x_3^2} , \qquad (94)$$

$$H^3 = z_1^3 x_1^3 + x_2^3 . \tag{89}$$

Differentiating H^3 with respect to θ_3^3 yields

$$\frac{\partial H^3}{\partial \theta_3^3} = z_1^3 \frac{\partial x_1^3}{\partial \theta_3^3} + \frac{\partial x_2^3}{\partial \theta_3^3} . \tag{90}$$

Setting $\frac{\partial H^3}{\partial \theta_3^3} = 0$ yields

$$z_1^3 = \frac{-\dfrac{\partial x_2^3}{\partial \theta_3^3}}{\dfrac{\partial x_1^3}{\partial \theta_3^3}} . \tag{91}$$

(c) Adjoint Variables z_i^n

z_i^N derived in the last section is used to calculate z_i^n in the following equations. In the actual calculation the values of the differentiation of the state variables in section (a) are substituted into the equation of z_i^n.

$$z_1^2 = z_1^3 \frac{\partial x_1^3}{\partial x_1^2} + z_2^3 \frac{\partial x_2^3}{\partial x_1^2} = z_1^3 \frac{\partial x_1^3}{\partial x_1^2} + \frac{\partial x_2^3}{\partial x_1^2} , \tag{92}$$

$$z_2^2 = z_1^3 \frac{\partial x_1^3}{\partial x_2^2} + z_2^3 \frac{\partial x_2^3}{\partial x_2^2}$$

$$= 1, \tag{93}$$

$$z_3^2 = z_1^3 \cdot \frac{\partial x_1^3}{\partial x_3^2} + z_2^3 \frac{\partial x_2^3}{\partial x_3^2} = \frac{\partial x_2^3}{\partial x_3^2} , \tag{94}$$

$$z_1^1 = z_1^2 \frac{\partial x_1^2}{\partial x_1^1} + z_2^2 \frac{\partial x_2^2}{\partial x_1^1} + z_3^2 \frac{\partial x_3^2}{\partial x_1^1}.$$

$$= z_1^2 \frac{\partial x_1^2}{\partial x_1^1} + \frac{\partial x_2^2}{\partial x_1^1} \quad , \tag{95}$$

$$z_2^1 = z_1^2 \frac{\partial x_1^2}{\partial x_2^1} + z_2^2 \frac{\partial x_2^2}{\partial x_2^1} + z_3^2 \frac{\partial x_3^2}{\partial x_2^1}$$

$$= 1, \tag{96}$$

$$z_3^1 = z_1^2 \frac{\partial x_1^2}{\partial x_3^1} + z_2^2 \frac{\partial x_2^2}{\partial x_3^1} + z_3^2 \frac{\partial x_3^2}{\partial x_3^1}$$

$$= \frac{\partial x_2^2}{\partial x_3^1} + z_3^2 \ , \tag{97}$$

(d) Derivatives of Hamiltonians

$$\frac{\partial H^1}{\partial \theta_1^1} = z_1^1 \frac{\partial x_1^1}{\partial \theta_1^1} + z_2^1 \frac{\partial x_2^1}{\partial \theta_1^1} + z_3^1 \frac{\partial x_3^1}{\partial \theta_1^1}$$

$$= z_1^1 \frac{\partial x_1^1}{\partial \theta_1^1} + \frac{\partial x_2^1}{\partial \theta_1^1} \ , \tag{98}$$

$$\frac{\partial H^1}{\partial \theta_3^1} = z_1^1 \frac{\partial x_1^1}{\partial \theta_3^1} + z_2^1 \frac{\partial x_2^1}{\partial \theta_3^1} + z_3^1 \frac{\partial x_3^1}{\partial \theta_3^1}$$

$$= z_1^1 \frac{\partial x_1^1}{\partial \theta_3^1} + \frac{\partial x_2^1}{\partial \theta_3^1} + z_3^1 \ , \tag{99}$$

$$z_1^1 = z_1^2 \frac{\partial x_1^2}{\partial x_1^1} + z_2^2 \frac{\partial x_2^2}{\partial x_1^1} + z_3^2 \frac{\partial x_3^2}{\partial x_1^1}.$$

$$= z_1^2 \frac{\partial x_1^2}{\partial x_1^1} + \frac{\partial x_2^2}{\partial x_1^1} \quad , \tag{95}$$

$$z_2^1 = z_1^2 \frac{\partial x_1^2}{\partial x_2^1} + z_2^2 \frac{\partial x_2^2}{\partial x_2^1} + z_3^2 \frac{\partial x_3^2}{\partial x_2^1}$$

$$= 1, \tag{96}$$

$$z_3^1 = z_1^2 \frac{\partial x_1^2}{\partial x_3^1} + z_2^2 \frac{\partial x_2^2}{\partial x_3^1} + z_3^2 \frac{\partial x_3^2}{\partial x_3^1}$$

$$= \frac{\partial x_2^2}{\partial x_3^1} + z_3^2 \; , \tag{97}$$

(d) Derivatives of Hamiltonians

$$\frac{\partial H^1}{\partial \theta_1^1} = z_1^1 \frac{\partial x_1^1}{\partial \theta_1^1} + z_2^1 \frac{\partial x_2^1}{\partial \theta_1^1} + z_3^1 \frac{\partial x_3^1}{\partial \theta_1^1}$$

$$= z_1^1 \frac{\partial x_1^1}{\partial \theta_1^1} + \frac{\partial x_2^1}{\partial \theta_1^1} \; , \tag{98}$$

$$\frac{\partial H^1}{\partial \theta_3^1} = z_1^1 \frac{\partial x_1^1}{\partial \theta_3^1} + z_2^1 \frac{\partial x_2^1}{\partial \theta_3^1} + z_3^1 \frac{\partial x_3^1}{\partial \theta_3^1}$$

$$= z_1^1 \frac{\partial x_1^1}{\partial \theta_3^1} + \frac{\partial x_2^1}{\partial \theta_3^1} + z_3^1 \; , \tag{99}$$

$$\frac{\partial H^2}{\partial \theta_1^2} = z_1^2 \frac{\partial x_1^2}{\partial \theta_1^2} + z_2^2 \frac{\partial x_2^2}{\partial \theta_1^2} + z_3^2 \frac{\partial x_3^2}{\partial \theta_1^2}$$

$$= z_1^2 \frac{\partial x_1^2}{\partial \theta_1^2} + \frac{\partial x_2^2}{\partial \theta_1^2} \quad , \qquad (100)$$

$$\frac{\partial H^2}{\partial \theta_3^2} = z_1^2 \frac{\partial x_1^2}{\partial \theta_3^2} + z_2^2 \frac{\partial x_2^2}{\partial \theta_3^2} + z_3^2 \frac{\partial x_3^2}{\partial \theta_3^2}$$

$$= z_1^2 \frac{\partial x_1^2}{\partial \theta_3^2} + \frac{\partial x_2^2}{\partial \theta_3^2} + z_3^2, \qquad (101)$$

$$\frac{\partial H^3}{\partial \theta_1^3} = z_1^3 \frac{\partial x_1^3}{\partial \theta_1^3} + z_2^3 \frac{\partial x_2^3}{\partial \theta_1^3} + z_3^3 \frac{\partial x_3^3}{\partial \theta_1^3}$$

$$= z_1^3 \frac{\partial x_1^3}{\partial \theta_1^3} + \frac{\partial x_2^3}{\partial \theta_1^3} \quad . \qquad (102)$$

(e) Calculation Procedures

Since x_1^3 and q_s/F are known, the optimal decisions θ_i^n for these fixed values of x_1^3 and q_s/F are obtained from the following procedures:

Step 1. Assume a set of values of θ_1^1, θ_1^2, θ_1^3, θ_3^1, θ_3^2, and $\Delta\theta_i^n$ as a trial.

Step 2. Calculate x_1^1, x_1^2, θ_3^3 from equation (60).

Step 3. Calculate x_3^1, x_3^2, x_3^3 from equation (59).

Step 4. Calculate x_2^0, x_2^1, x_2^2, x_2^3 from equations (61) through (66).

$$\frac{\partial H^2}{\partial \theta_1^2} = z_1^2 \frac{\partial x_1^2}{\partial \theta_1^2} + z_2^2 \frac{\partial x_2^2}{\partial \theta_1^2} + z_3^2 \frac{\partial x_3^2}{\partial \theta_1^2}$$

$$= z_1^2 \frac{\partial x_1^2}{\partial \theta_1^2} + \frac{\partial x_2^2}{\partial \theta_1^2} \quad , \tag{100}$$

$$\frac{\partial H^2}{\partial \theta_3^2} = z_1^2 \frac{\partial x_1^2}{\partial \theta_3^2} + z_2^2 \frac{\partial x_2^2}{\partial \theta_3^2} + z_3^2 \frac{\partial x_3^2}{\partial \theta_3^2}$$

$$= z_1^2 \frac{\partial x_1^2}{\partial \theta_3^2} + \frac{\partial x_2^2}{\partial \theta_3^2} + z_3^2, \tag{101}$$

$$\frac{\partial H^3}{\partial \theta_1^3} = z_1^3 \frac{\partial x_1^3}{\partial \theta_1^3} + z_2^3 \frac{\partial x_2^3}{\partial \theta_1^3} + z_3^3 \frac{\partial x_3^3}{\partial \theta_1^3}$$

$$= z_1^3 \frac{\partial x_1^3}{\partial \theta_1^3} + \frac{\partial x_2^3}{\partial \theta_1^3} \quad . \tag{102}$$

(e) Calculation Procedures

Since x_1^3 and q_s/F are known, the optimal decisions θ_i^n for these fixed values of x_1^3 and q_s/F are obtained from the following procedures:

Step 1. Assume a set of values of θ_1^1, θ_1^2, θ_1^3, θ_3^1, θ_3^2, and $\Delta\theta_i^n$ as a trial.

Step 2. Calculate x_1^1, x_1^2, θ_3^3 from equation (60).

Step 3. Calculate x_3^1, x_3^2, x_3^3 from equation (59).

Step 4. Calculate x_2^0, x_2^1, x_2^2, x_2^3 from equations (61) through (66).

Step 5. Calculate z_1^3 from equation (91) and equations (68) and (78), and calculate z_1^2, z_2^2, z_3^2, z_1^1, z_2^1, z_3^1 from equations (92) through (97).

Step 6. Calculate $\frac{\partial H^1}{\partial \theta_1^1}$ $\frac{\partial H^1}{\partial \theta_3^1}$ $\frac{\partial H^2}{\partial \theta_1^2}$ $\frac{\partial H^2}{\partial \theta_3^2}$ $\frac{\partial H^3}{\partial \theta_1^3}$ from equations (98) through (102).

Step 7. If $\frac{\partial H^n}{\partial \theta_i^n}$ are zero or less than the allowable errors preassigned, then the assumed θ_i^n are the optimal values; otherwise go to the next step.

Step 8. If x_2^3 is greater than that computed in the preceding iteration, then one half of the original $\Delta\theta_i^n$ is used; otherwise the original $\Delta\theta_i^n$ is used.

Step 9. The new set of decision variables $(\theta_i^n)_{new}$ is obtained by

$$(\theta_i^n)_{new} = (\theta_i^n)_{old} \pm \Delta\theta_i^n \tag{103}$$

When

$$\frac{\partial H^n}{\partial \theta_i^n} > 0 \quad \text{use (-) sign}$$

When

$$\frac{\partial H^n}{\partial \theta_i^n} < 0 \quad \text{use (+) sign}$$

Then go to step 2 and repeat the computation until the optimum is obtained.

(f) Computer Flow Chart

The numerical values of the various constants involved in the performance equations are summarized in Table 1. These values are

Step 5. Calculate z_1^3 from equation (91) and equations (68) and (78), and calculate z_1^2, z_2^2, z_3^2, z_1^1, z_2^1, z_3^1 from equations (92) through (97).

Step 6. Calculate $\frac{\partial H^1}{\partial \theta_1^1}$ $\frac{\partial H^1}{\partial \theta_3^1}$ $\frac{\partial H^2}{\partial \theta_1^2}$ $\frac{\partial H^2}{\partial \theta_3^2}$ $\frac{\partial H^3}{\partial \theta_1^3}$ from equations (98) through (102).

Step 7. If $\frac{\partial H^n}{\partial \theta_i^n}$ are zero or less than the allowable errors preassigned, then the assumed θ_i^n are the optimal values; otherwise go to the next step.

Step 8. If x_2^3 is greater than that computed in the preceding iteration, then one half of the original $\Delta\theta_i^n$ is used; otherwise the original $\Delta\theta_i^n$ is used.

Step 9. The new set of decision variables $(\theta_i^n)_{new}$ is obtained by

$$(\theta_i^n)_{new} = (\theta_i^n)_{old} \pm \Delta\theta_i^n \tag{103}$$

When

$$\frac{\partial H^n}{\partial \theta_i^n} > 0 \quad \text{use (-) sign}$$

When

$$\frac{\partial H^n}{\partial \theta_i^n} < 0 \quad \text{use (+) sign}$$

Then go to step 2 and repeat the computation until the optimum is obtained.

(f) Computer Flow Chart

The numerical values of the various constants involved in the performance equations are summarized in Table 1. These values are

taken from references (10) and (12). A computer flow chart based on the procedure we have described in part (e) is given Fig. 7.

Table 1. Numerical Values for the Constants

Symbols	Explanation	Numerical values
a or T_s	Steam temperature	274.4^oF
B	Coefficient of the Clausieus-Clapeyron equation	1.79×10^9 lb_f/ft^2
C_c	Unit cost of cooling water	4.4875×10^{-7} \$/lb
C_F	Concentration of sea water feed	0.035 wt. fraction
C_{ht}	Unit cost of brine heater	3.76×10^{-5} \$/ft^2
C_p	Heat capacity of sea water	1.0 Btu/lb^oF
C_s	Unit cost of steam	2.5×10^{-4} \$/lb
C_{cd}	Unit cost of condensing area	2.397×10^{-5} \$/ft^2
C_{pp}	Unit cost of pump and pumping power	2.903×10^{-9} \$/ft-lb
Pc	Unit cost of feed pretreatment	1.1795×10^{-6} \$/lb
N^n	No. of stages in n-th effect	23, 23, 22
U	Overall heat transfer coefficient	510 Btu/hr. ft^{2o}F
λ	Latent heat of flash brine	1000 Btu/lb
λ_s	Latent heat of steam at 274.4^oF and 45 psia	928.9 Btu/lb
R	Ideal gas constant	0.1104 Btu/lb.^oF
ρ	Density of brine	62.5 lb/ft^3
x_3^0	Temperature of brine entering the first effect	250^oF

taken from references (10) and (12). A computer flow chart based on the procedure we have described in part (e) is given Fig. 7.

Table 1. Numerical Values for the Constants

Symbols	Explanation	Numerical values
a or T_s	Steam temperature	274.4°F
B	Coefficient of the Clausieus-Clapeyron equation	1.79×10^9 lb_f/ft^2
C_c	Unit cost of cooling water	4.4875×10^{-7} \$/lb
C_F	Concentration of sea water feed	0.035 wt. fraction
C_{ht}	Unit cost of brine heater	3.76×10^{-5} \$/ft^2
C_p	Heat capacity of sea water	1.0 Btu/lb°F
C_s	Unit cost of steam	2.5×10^{-4} \$/lb
C_{cd}	Unit cost of condensing area	2.397×10^{-5} \$/ft^2
C_{pp}	Unit cost of pump and pumping power	2.903×10^{-9} \$/ft-lb
Pc	Unit cost of feed pretreatment	1.1795×10^{-6} \$/lb
N^n	No. of stages in n-th effect	23, 23, 22
U	Overall heat transfer coefficient	510 Btu/hr. ft^2°F
λ	Latent heat of flash brine	1000 Btu/lb
λ_s	Latent heat of steam at 274.4°F and 45 psia	928.9 Btu/lb
R	Ideal gas constant	0.1104 Btu/lb.°F
ρ	Density of brine	62.5 lb/ft^3
x_3^0	Temperature of brine entering the first effect	250°F

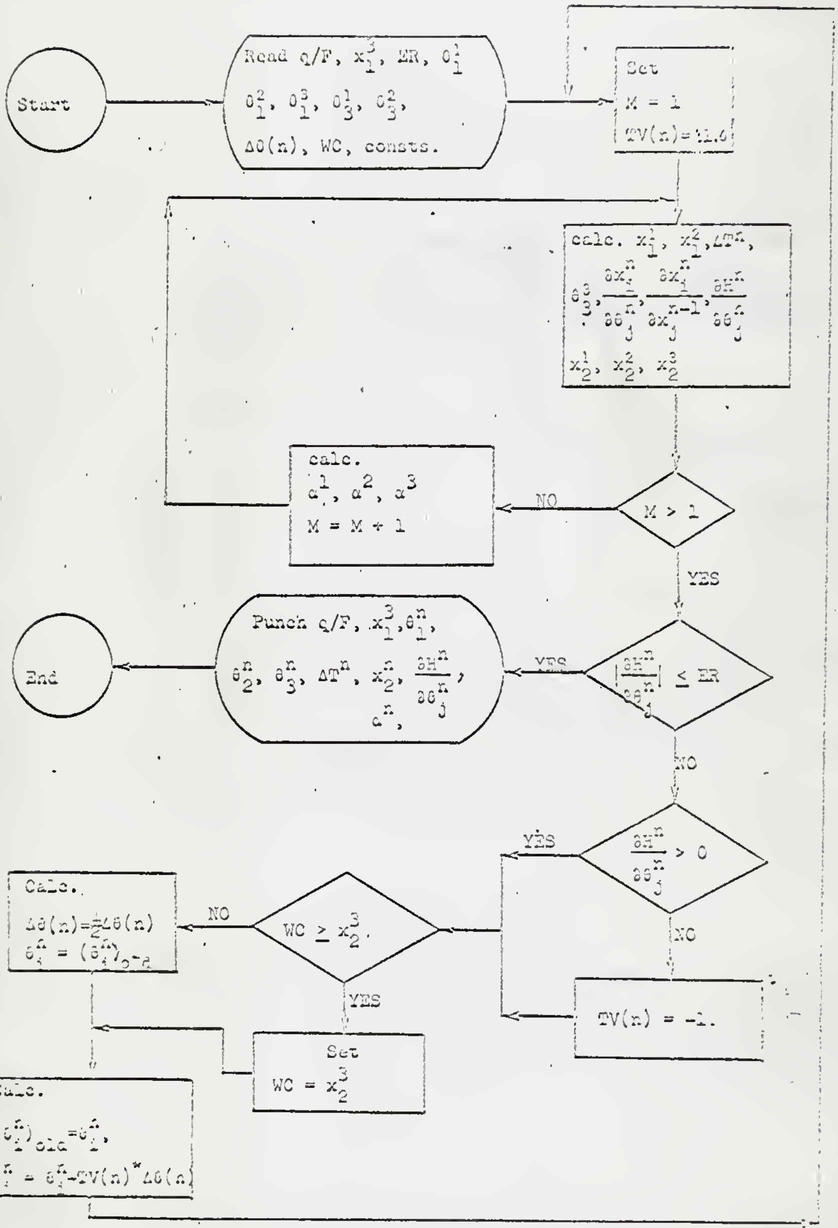


Fig. 7. Computer flow diagram.

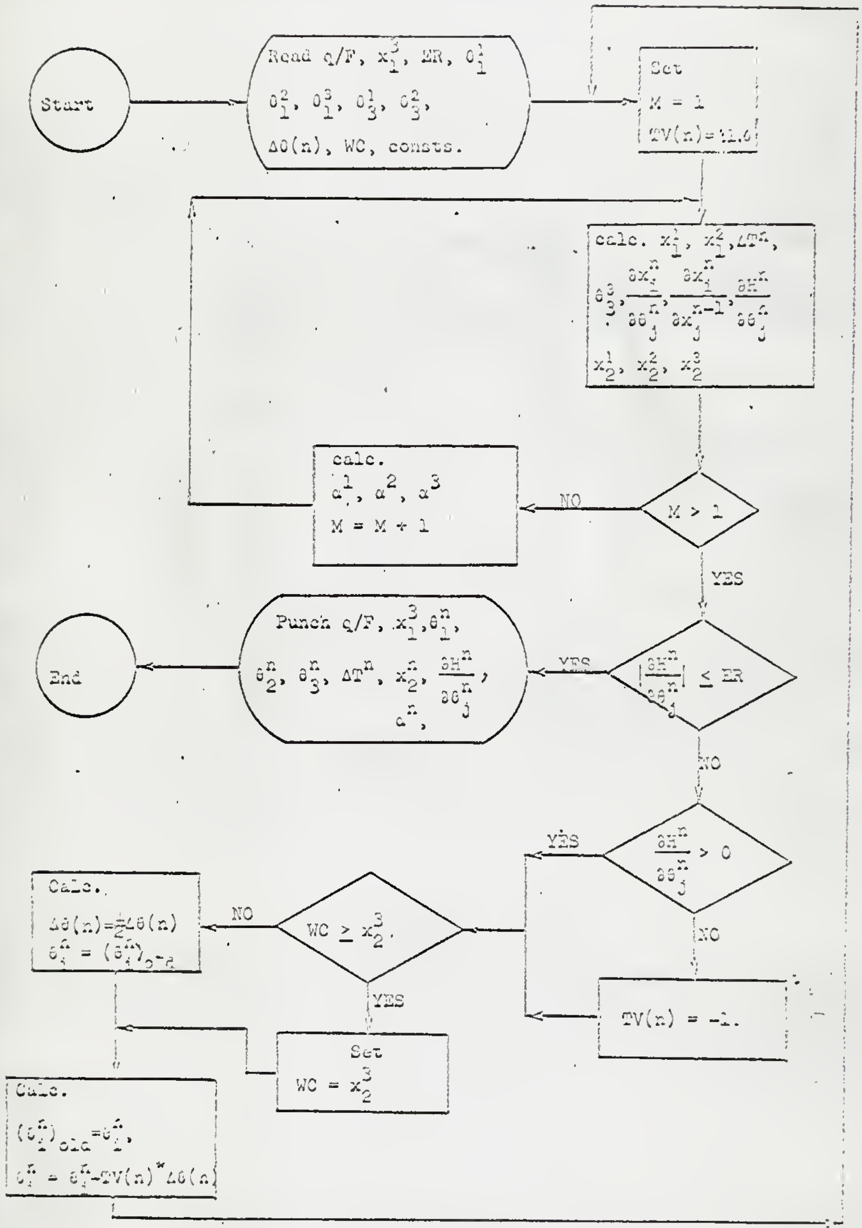


Fig. 7. Computer flow diagram.

5-3. Parametric Search for Overall Optimum

A set of grid points of x_1^3 and q_s/F is shown in Table 2. For each grid point the scheme we have described in the last section was used to seek the optimum. Since corresponding to a given x_1^3 we have a set of q_s/F, we can use a graphical method to find an overall optimum for a given x_1^3. These data are given in Table 3 and the corresponding figures are shown in Figs. 8 through 15. The optimal policies for each x_1^3 are plotted in Fig. 16, from which the optimal condition of the whole system was obtained.

In Table 3, for given x_1^3 and q_s/F, the optimal values of r_n and $(T_f)_n$, and the water cost, C, are tabulated. For example, for $x_1^3 = 0.05$ and $q_s/F = 16$, the optimal policy is

$$r_1 = 0.99 \quad r_2 = 1.30 \quad r_3 = 1.72$$

$$(T_f)_1 = 198^{\circ}F \quad (T_f)_2 = 151^{\circ}F \quad (T_f)_3 = 102^{\circ}F$$

$$C = 0.2907\$/1000 \text{ gal.}$$

In Figs. 8 through 15, these optimal policies are plotted against q_s/F for a fixed x_1^3.

From Fig. 16, the overall minimum water production cost is found to be 0.2855$/1000 gal., when the system is operated under the following conditions:

salt concentration of the flashing brine leaving the third effect,	$x_1^3 = 0.065$,
the ratio of heat load to seawater feed	$q_s/F = 27$,
recycle ratio in the first effect,	$r_1 = 2.14$,
recycle ratio in the second effect	$r_2 = 2.88$,
recycle ratio in the third effect,	$r_3 = 3.86$,

5-3. Parametric Search for Overall Optimum

A set of grid points of x_1^3 and q_s/F is shown in Table 2. For each grid point the scheme we have described in the last section was used to seek the optimum. Since corresponding to a given x_1^3 we have a set of q_s/F, we can use a graphical method to find an overall optimum for a given x_1^3. These data are given in Table 3 and the corresponding figures are shown in Figs. 8 through 15. The optimal policies for each x_1^3 are plotted in Fig. 16, from which the optimal condition of the whole system was obtained.

In Table 3, for given x_1^3 and q_s/F, the optimal values of r_n and $(T_f)_n$, and the water cost, C, are tabulated. For example, for $x_1^3 = 0.05$ and $q_s/F = 16$, the optimal policy is

$$r_1 = 0.99 \quad r_2 = 1.30 \quad r_3 = 1.72$$

$$(T_f)_1 = 198^{\circ}F \quad (T_f)_2 = 151^{\circ}F \quad (T_f)_3 = 102^{\circ}F$$

$$C = 0.2907\$/1000 \text{ gal.}$$

In Figs. 8 through 15, these optimal policies are plotted against q_s/F for a fixed x_1^3.

From Fig. 16, the overall minimum water production cost is found to be 0.2855\$/1000 gal., when the system is operated under the following conditions:

salt concentration of the flashing brine leaving the third effect,	$x_1^3 = 0.065$,
the ratio of heat load to seawater feed	$q_s/F = 27$,
recycle ratio in the first effect,	$r_1 = 2.14$,
recycle ratio in the second effect	$r_2 = 2.88$,
recycle ratio in the third effect,	$r_3 = 3.86$,

Table 2. Grid Points for Parametric Search

q_s/F \ x_1^3	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12
14	X							
16	X							
17	X							
18	X							
20	X	X						
23		X						
24		X						
25		X	X					
28			X	X				
29			X					
30		X	X		X			
32			X	X				
33				X				
34				X		X		
35			X	X	X		X	
36					X			
37					X			X
38				X	X	X		
39						X		
40						X	X	
41						X	X	
42							X	
43							X	X
44								X
45					X	X		X
46								X
48							X	
50								X

Table 2. Grid Points for Parametric Search

$q_{s/F}$ \ x_1^3	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12
14	X							
16	X							
17	X							
18	X							
20	X	X						
23		X						
24		X						
25		X	X					
28			X	X				
29			X					
30		X	X		X			
32			X	X				
33				X				
34				X		X		
35			X	X	X		X	
36					X			
37					X			X
38				X	X	X		
39						X		
40						X	X	
41						X	X	
42							X	
43							X	X
44								X
45					X	X		X
46								X
48							X	
50								X

Table 3.

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_s/F

q_s/F \ x_1^3	0.05	0.06
14	$r_1=0.96, r_2=1.26, r_3=1.65$ $(T_f)_1=197, (T_f)_2=149, (T_f)_3=99$ C=0.2985 \$/1000 gal.	
16	$r_1=0.99, r_2=1.30, r_3=1.72$ $(T_f)_1=198, (T_f)_2=151, (T_f)_3=102$ C=0.2907 \$/1000 gal.	
17	$r_1=1.00, r_2=1.32, r_3=1.75$ $(T_f)_1=199, (T_f)_2=152, (T_f)_3=104$ C=0.2900 \$/1000 gal.	
18	$r_1=1.01, r_2=1.35, r_3=1.79$ $(T_f)_1=199, (T_f)_2=152, (T_f)_3=105$ C=0.2908 \$/1000 gal.	
20	$r_1=1.02, r_2=1.39, r_3=1.87$ $(T_f)_1=199, (T_f)_2=153, (T_f)_3=107$ C=0.2954 \$/1000 gal.	$r_1=1.79, r_2=2.29, r_3=3.05$ $(T_f)_1=197, (T_f)_2=147, (T_f)_3=99$ C=0.2941 \$/1000 gal.
23		$r_1=1.81, r_2=2.37, r_3=3.17$ $(T_f)_1=197, (T_f)_2=148, (T_f)_3=102$ C=0.2863 \$/1000 gal.
24		$r_1=1.82, r_2=2.41, r_3=3.20$ $(T_f)_1=197, (T_f)_2=149, (T_f)_3=103$ C=0.2858 \$/1000 gal.
25		$r_1=1.83, r_2=2.42, r_3=3.24$ $(T_f)_1=198, (T_f)_2=150, (T_f)_3=104$ C=0.2862 \$/1000 gal.
30		$r_1=1.36, r_2=2.56, r_3=3.44$ $(T_f)_1=198, (T_f)_2=152, (T_f)_3=109$ C=0.2955 \$/1000 gal.

Table 3.

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_s/F

q_s/F \ x_1^3	0.05	0.06
14	$r_1=0.96, r_2=1.26, r_3=1.65$ $(T_r)_1=197, (T_r)_2=149, (T_r)_3=99$ C=0.2985 \$/1000 gal.	
16	$r_1=0.99, r_2=1.30, r_3=1.72$ $(T_r)_1=198, (T_r)_2=151, (T_r)_3=102$ C=0.2907 \$/1000 gal.	
17	$r_1=1.00, r_2=1.32, r_3=1.75$ $(T_r)_1=199, (T_r)_2=152, (T_r)_3=104$ C=0.2900 \$/1000 gal.	
18	$r_1=1.01, r_2=1.35, r_3=1.79$ $(T_r)_1=199, (T_r)_2=152, (T_r)_3=105$ C=0.2908 \$/1000 gal.	
20	$r_1=1.02, r_2=1.39, r_3=1.87$ $(T_r)_1=199, (T_r)_2=153, (T_r)_3=107$ C=0.2954 \$/1000 gal.	$r_1=1.79, r_2=2.29, r_3=3.05$ $(T_r)_1=197, (T_r)_2=147, (T_r)_3=99$ C=0.2941 \$/1000 gal.
23		$r_1=1.81, r_2=2.37, r_3=3.17$ $(T_r)_1=197, (T_r)_2=148, (T_r)_3=102$ C=0.2863 \$/1000 gal.
24		$r_1=1.82, r_2=2.41, r_3=3.20$ $(T_r)_1=197, (T_r)_2=149, (T_r)_3=103$ C=0.2858 \$/1000 gal.
25		$r_1=1.83, r_2=2.42, r_3=3.24$ $(T_r)_1=198, (T_r)_2=150, (T_r)_3=104$ C=0.2862 \$/1000 gal.
30		$r_1=1.36, r_2=2.56, r_3=3.44$ $(T_r)_1=198, (T_r)_2=152, (T_r)_3=109$ C=0.2955 \$/1000 gal.

Table 3. (Continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1^5 and q_s/F

q_s/F \ x_1^5	0.07	0.08
.25	$r_1=2.40, r_2=3.16, r_3=4.23$ $(T_f)_1=196, (T_f)_2=146, (T_f)_3=100$ C=0.2924 $/1000 gal.	
28	$r_1=2.41, r_2=3.26, r_3=4.40$ $(T_f)_1=195, (T_f)_2=147, (T_f)_3=102$ C=0.2863 $/100 gal.	$r_1=2.87, r_2=3.85, r_3=5.25$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=99$ C=0.2961 $/1000 gal.
29	$r_1=2.43, r_2=3.28, r_3=4.43$ $(T_f)_1=196, (T_f)_2=147, (T_f)_3=103$ C=0.2857 $/1000 gal.	
30	$r_1=2.44, r_2=3.31, r_3=4.45$ $(T_f)_1=196, (T_f)_2=148, (T_f)_3=104$ C=0.2858 $/1000 gal.	
32	$r_1=2.47, r_2=3.35, r_3=4.49$ $(T_f)_1=197, (T_f)_2=150, (T_f)_3=106$ C=0.2872 $/1000 gal.	$r_1=2.93, r_2=3.93, r_3=5.39$ $(T_f)_1=197, (T_f)_2=146, (T_f)_3=102$ C=0.2874 $/1000 gal.
33		$r_1=2.94, r_2=3.95, r_3=5.46$ $(T_f)_1=197, (T_f)_2=146, (T_f)_3=103$ C=0.2869 $/1000 gal.
34		$r_1=2.95, r_2=3.97, r_3=5.52$ $(T_f)_1=197, (T_f)_2=147, (T_f)_3=104$ C=0.2868 $/1000 gal.
35	$r_1=2.49, r_2=3.42, r_3=4.64$ $(T_f)_1=198, (T_f)_2=151, (T_f)_3=108$ C=0.2919 $/1000 gal.	$r_1=2.96, r_2=3.40, r_3=5.57$ $(T_f)_1=197, (T_f)_2=147, (T_f)_3=105$ C=0.2872 $/1000 gal.
38		$r_1=2.95, r_2=4.17, r_3=5.73$ $(T_f)_1=195, (T_f)_2=148, (T_f)_3=107$ C=0.2900 $/1000 gal.

Table 3. (Continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1° and q_s/F

q_s/F \ x_1°	0.07	0.08
25	$r_1=2.40, r_2=3.16, r_3=4.23$ $(T_f)_1=196, (T_f)_2=146, (T_f)_3=100$ C=0.2924 \$/1000 gal.	
28	$r_1=2.41, r_2=3.26, r_3=4.40$ $(T_f)_1=195, (T_f)_2=147, (T_f)_3=102$ C=0.2863 \$/100 gal.	$r_1=2.87, r_2=3.85, r_3=5.25$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=99$ C=0.2961 \$/1000 gal.
29	$r_1=2.43, r_2=3.28, r_3=4.43$ $(T_f)_1=196, (T_f)_2=147, (T_f)_3=103$ C=0.2857 \$/1000 gal.	
30	$r_1=2.44, r_2=3.31, r_3=4.45$ $(T_f)_1=196, (T_f)_2=148, (T_f)_3=104$ C=0.2858 \$/1000 gal.	
32	$r_1=2.47, r_2=3.35, r_3=4.49$ $(T_f)_1=197, (T_f)_2=150, (T_f)_3=106$ C=0.2872 \$/1000 gal.	$r_1=2.93, r_2=3.93, r_3=5.39$ $(T_f)_1=197, (T_f)_2=146, (T_f)_3=102$ C=0.2874 \$/1000 gal.
33		$r_1=2.94, r_2=3.95, r_3=5.46$ $(T_f)_1=197, (T_f)_2=146, (T_f)_3=103$ C=0.2869 \$/1000 gal.
34		$r_1=2.95, r_2=3.97, r_3=5.52$ $(T_f)_1=197, (T_f)_2=147, (T_f)_3=104$ C=0.2868 \$/1000 gal.
35	$r_1=2.49, r_2=3.42, r_3=4.64$ $(T_f)_1=198, (T_f)_2=151, (T_f)_3=108$ C=0.2919 \$/1000 gal.	$r_1=2.96, r_2=3.40, r_3=5.57$ $(T_f)_1=197, (T_f)_2=147, (T_f)_3=105$ C=0.2872 \$/1000 gal.
38		$r_1=2.95, r_2=4.17, r_3=5.73$ $(T_f)_1=195, (T_f)_2=148, (T_f)_3=107$ C=0.2900 \$/1000 gal.

Table 3 (continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_x/F

q_x/F \ x_1^3	0.09	0.10
30	$r_1=3.24, r_2=4.44, r_3=6.14$ $(T_f)_1=192, (T_f)_2=142, (T_f)_3=98$ C=0.3021 \$/1000 gal.	
34		$r_1=3.56, r_2=5.05, r_3=7.12$ $(T_f)_1=191, (T_f)_2=140, (T_f)_3=100$ C=0.2986 \$/1000 gal.
35	$r_1=3.30, r_2=4.57, r_3=6.38$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=102$ C=0.2895 \$/1000 gal.	
36	$r_1=3.29, r_2=4.65, r_3=6.50$ $(T_f)_1=192, (T_f)_2=143, (T_f)_3=103$ C=0.2889 \$/1000 gal.	
37	$r_1=3.33, r_2=4.60, r_3=6.48$ $(T_f)_1=195, (T_f)_2=144, (T_f)_3=103$ C=0.2885 \$/1000 gal.	
38	$r_1=3.33, r_2=4.64, r_3=6.59$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=104$ C=0.2888 \$/1000 gal.	$r_1=3.61, r_2=5.17, r_3=7.35$ $(T_f)_1=191, (T_f)_2=141, (T_f)_3=102$ C=0.2913 \$/1000 gal.
39		$r_1=3.62, r_2=5.20, r_3=7.43$ $(T_f)_1=191, (T_f)_2=141, (T_f)_3=103$ C=0.2908 \$/1000 gal.
40		$r_1=3.60, r_2=5.31, r_3=7.60$ $(T_f)_1=189, (T_f)_2=141, (T_f)_3=103$ C=0.2907 \$/1000 gal.
41		$r_1=3.61, r_2=5.36, r_3=7.60$ $(T_f)_1=189, (T_f)_2=142, (T_f)_3=104$ C=0.2909 \$/1000 gal.
45	$r_1=3.36, r_2=4.89, r_3=6.97$ $(T_f)_1=194, (T_f)_2=147, (T_f)_3=108$ C=0.2962 \$/1000 gal.	$r_1=3.63, r_2=5.52, r_3=7.89$ $(T_f)_1=189, (T_f)_2=142, (T_f)_3=106$ C=0.2934 \$/1000 gal.

Table 3 (continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_x/F

q_x/F \ x_1^3	0.09	0.10
30	$r_1=3.24, r_2=4.44, r_3=6.14$ $(T_f)_1=192, (T_f)_2=142, (T_f)_3=98$ C=0.3021 $/1000 gal.	
34		$r_1=3.56, r_2=5.05, r_3=7.12$ $(T_f)_1=191, (T_f)_2=140; (T_f)_3=100$ C=0.2986 $/1000 gal.
35	$r_1=3.30, r_2=4.57, r_3=6.38$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=102$ C=0.2895 $/1000 gal.	
36	$r_1=3.29, r_2=4.65, r_3=6.50$ $(T_f)_1=192, (T_f)_2=143, (T_f)_3=103$ C=0.2889 $/1000 gal.	
37	$r_1=3.33, r_2=4.60, r_3=6.48$ $(T_f)_1=195, (T_f)_2=144, (T_f)_3=103$ C=0.2885 $/1000 gal.	
38	$r_1=3.33, r_2=4.64, r_3=6.59$ $(T_f)_1=194, (T_f)_2=144, (T_f)_3=104$ C=0.2888 $/1000 gal.	$r_1=3.61, r_2=5.17, r_3=7.35$ $(T_f)_1=191, (T_f)_2=141, (T_f)_3=102$ C=0.2913 $/1000 gal.
39		$r_1=3.62, r_2=5.20, r_3=7.43$ $(T_f)_1=191, (T_f)_2=141, (T_f)_3=103$ C=0.2908 $/1000 gal.
40		$r_1=3.60, r_2=5.31, r_3=7.60$ $(T_f)_1=189, (T_f)_2=141, (T_f)_3=103$ C=0.2907 $/1000 gal.
41		$r_1=3.61, r_2=5.36, r_3=7.60$ $(T_f)_1=189, (T_f)_2=142, (T_f)_3=104$ C=0.2909 $/1000 gal.
45	$r_1=3.36, r_2=4.89, r_3=6.97$ $(T_f)_1=194, (T_f)_2=147, (T_f)_3=108$ C=0.2962 $/1000 gal.	$r_1=3.63, r_2=5.52, r_3=7.89$ $(T_f)_1=189, (T_f)_2=142, (T_f)_3=106$ C=0.2934 $/1000 gal.

Table 3 (continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_x/F

q_x/F \ x_1^3	0.11	0.12
35	$r_1=3.80, r_2=5.59, r_3=7.93$ $(T_f)_1=187, (T_f)_2=137, (T_f)_3=99$ C=0.3055 \$/1000 gal.	
37		$r_1=4.07, r_2=5.94, r_3=8.69$ $(T_f)_1=188, (T_f)_2=136, (T_f)_3=99$ C=0.3073 \$/1000 gal.
40	$r_1=3.85, r_2=5.75, r_3=8.34$ $(T_f)_1=187, (T_f)_2=138, (T_f)_3=102$ C=0.2941 \$/1000 gal.	
41	$r_1=3.87, r_2=5.79, r_3=8.31$ $(T_f)_1=188, (T_f)_2=139, (T_f)_3=103$ C=0.2932 \$/1000 gal.	
42	$r_1=3.86, r_2=5.83, r_3=8.50$ $(T_f)_1=187, (T_f)_2=138, (T_f)_3=103$ C=0.2929 \$/1000 gal.	
43	$r_1=3.88, r_2=5.82, r_3=8.59$ $(T_f)_1=188, (T_f)_2=138, (T_f)_3=104$ C=0.2931 \$/1000 gal.	$r_1=4.11, r_2=6.18, r_3=9.35$ $(T_f)_1=187, (T_f)_2=136, (T_f)_3=102$ C=0.2953 \$/1000 gal.
44		$r_1=4.12, r_2=6.22, r_3=9.39$ $(T_f)_1=187, (T_f)_2=136, (T_f)_3=103$ C=0.2948 \$/1000 gal.
45		$r_1=4.14, r_2=6.22, r_3=9.41$ $(T_f)_1=188, (T_f)_2=137, (T_f)_3=104$ C=0.2945 \$/1000 gal.
46		$r_1=4.15, r_2=6.25, r_3=9.50$ $(T_f)_1=187, (T_f)_2=137, (T_f)_3=104$ C=0.2952 \$/1000 gal.
48	$r_1=3.93, r_2=6.01, r_3=8.94$ $(T_f)_1=188, (T_f)_2=140, (T_f)_3=107$ C=0.2956 \$/1000 gal.	
50		$r_1=4.17, r_2=6.41, r_3=9.71$ $(T_f)_1=188, (T_f)_2=139, (T_f)_3=106$ C=0.2970 \$/1000 gal.

Table 3 (continued)

THE OPTIMUM POLICIES FOR VARIOUS x_1^3 and q_x/F

q_x/F \ x_1^3	0.11	0.12
35	$r_1=3.80, r_2=5.59, r_3=7.93$ $(T_f)_1=187, (T_f)_2=137, (T_f)_3=99$ C=0.3055 \$/1000 gal.	
37		$r_1=4.07, r_2=5.94, r_3=8.69$ $(T_f)_1=188, (T_f)_2=136, (T_f)_3=99$ C=0.3073 \$/1000 gal.
40	$r_1=3.85, r_2=5.75, r_3=8.34$ $(T_f)_1=187, (T_f)_2=138, (T_f)_3=102$ C=0.2941 \$/1000 gal.	
41	$r_1=3.87, r_2=5.79, r_3=8.31$ $(T_f)_1=188, (T_f)_2=139, (T_f)_3=103$ C=0.2932 \$/1000 gal.	
42	$r_1=3.86, r_2=5.83, r_3=8.50$ $(T_f)_1=187, (T_f)_2=138, (T_f)_3=103$ C=0.2929 \$/1000 gal.	
43	$r_1=3.88, r_2=5.82, r_3=8.59$ $(T_f)_1=188, (T_f)_2=138, (T_f)_3=104$ C=0.2931 \$/1000 gal.	$r_1=4.11, r_2=6.18, r_3=9.35$ $(T_f)_1=187, (T_f)_2=136, (T_f)_3=102$ C=0.2953 \$/1000 gal.
44		$r_1=4.12,\ r_2=6.22,\ r_3=9.39$ $(T_f)_1=187, (T_f)_2=136, (T_f)_3=103$ C=0.2948 \$/1000 gal.
45		$r_1=4.14, r_2=6.22, r_3=9.41$ $(T_f)_1=188, (T_f)_2=137, (T_f)_3=104$ C=0.2945 \$/1000 gal.
46		$r_1=4.15, r_2=6.25, r_3=9.50$ $(T_f)_1=187, (T_f)_2=137, (T_f)_3=104$ C=0.2952 \$/1000 gal.
48	$r_1=3.93, r_2=6.01, r_3=8.94$ $(T_f)_1=188, (T_f)_2=140, (T_f)_3=107$ C=0.2956 \$/1000 gal.	
50		$r_1=4.17, r_2=6.41, r_3=9.71$ $(T_f)_1=188, (T_f)_2=139, (T_f)_3=106$ C=0.2970 \$/1000 gal.

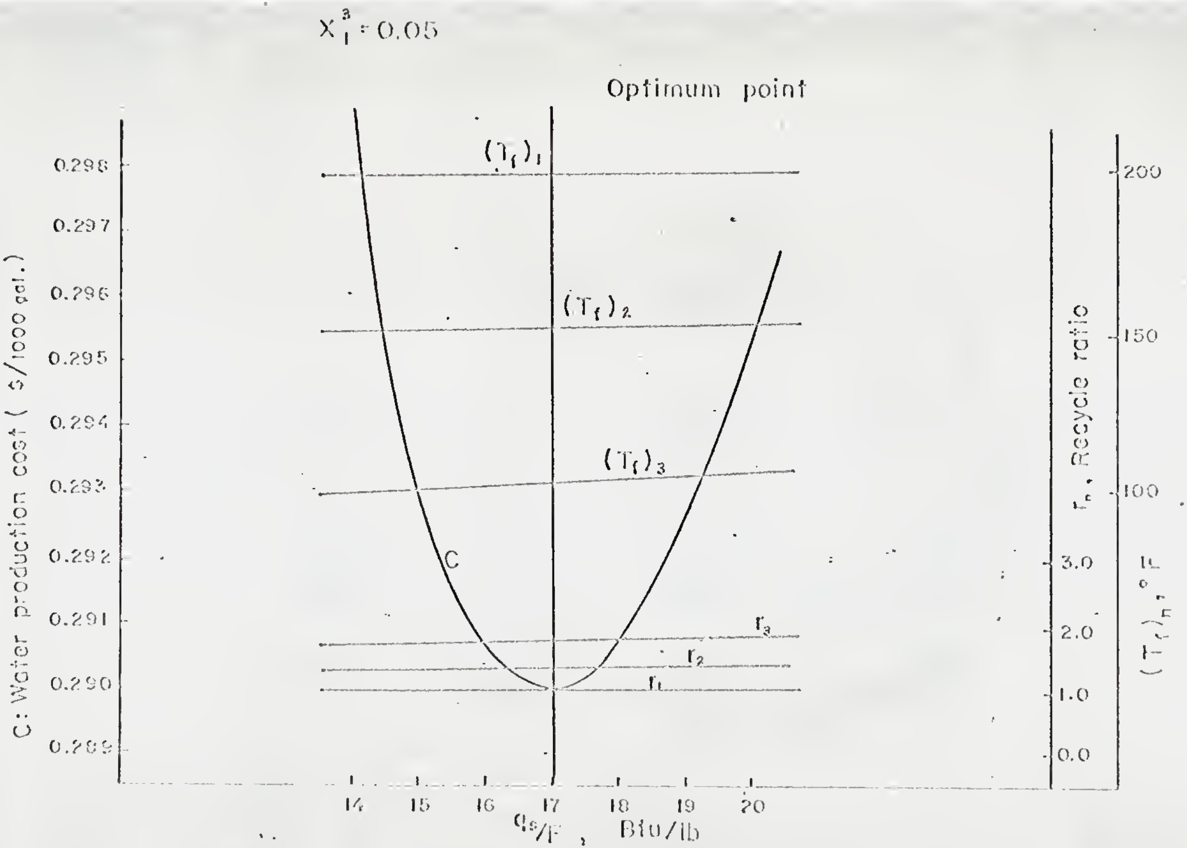


Fig. 8. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 5\%$.

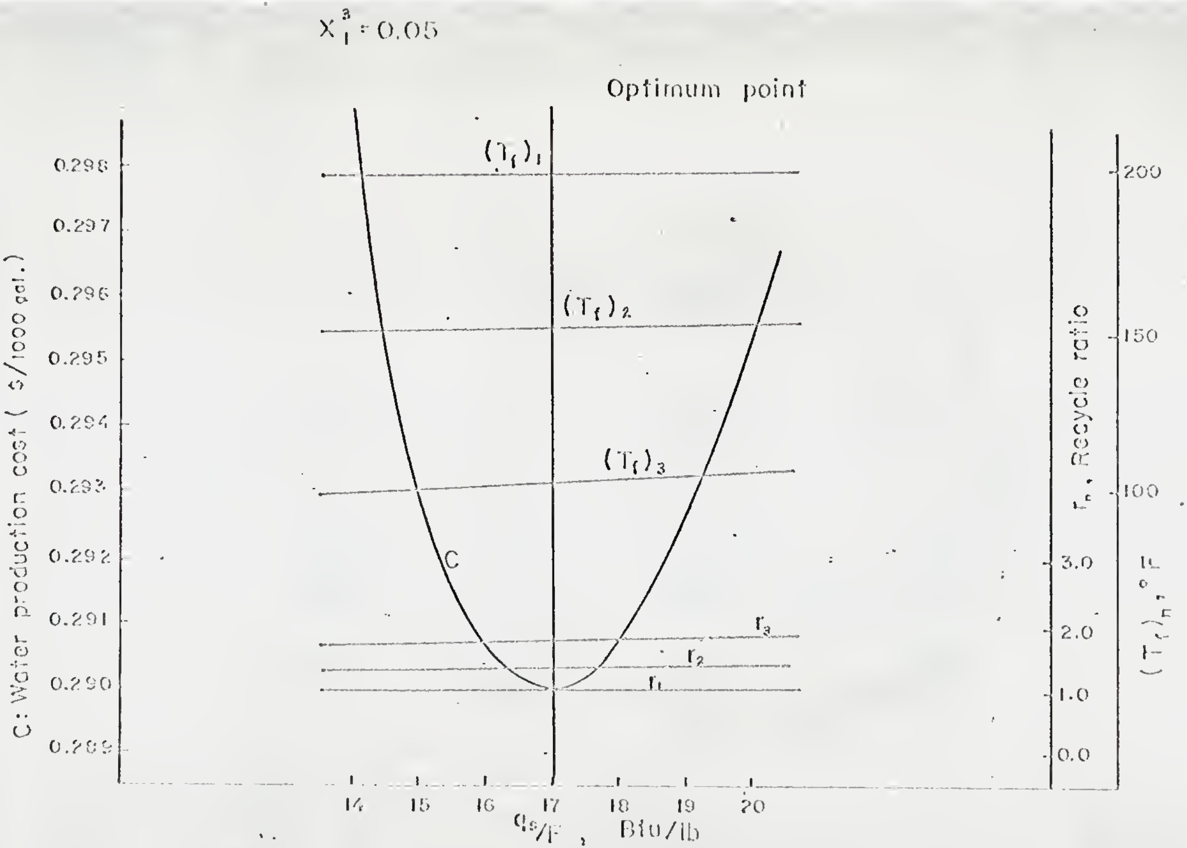


Fig. 8. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 5\%$.

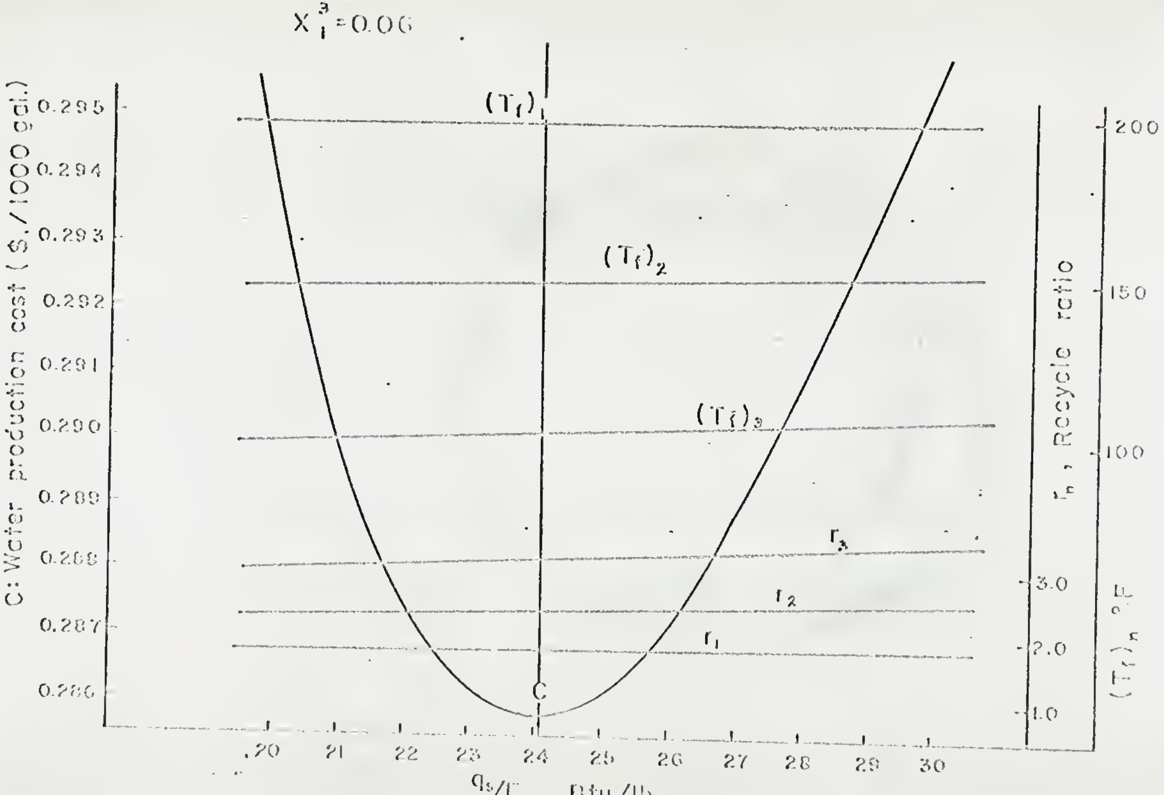


Fig.9. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 6\ \%$.

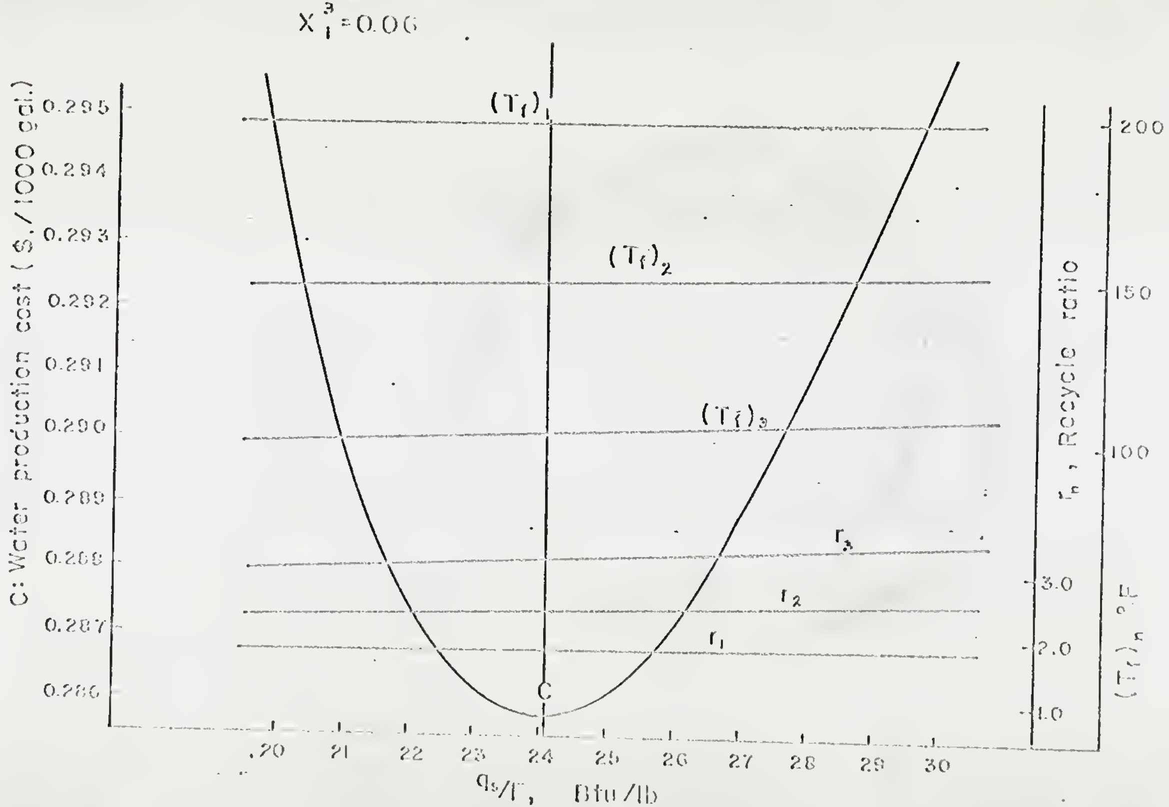


Fig. 9. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 6\%$.

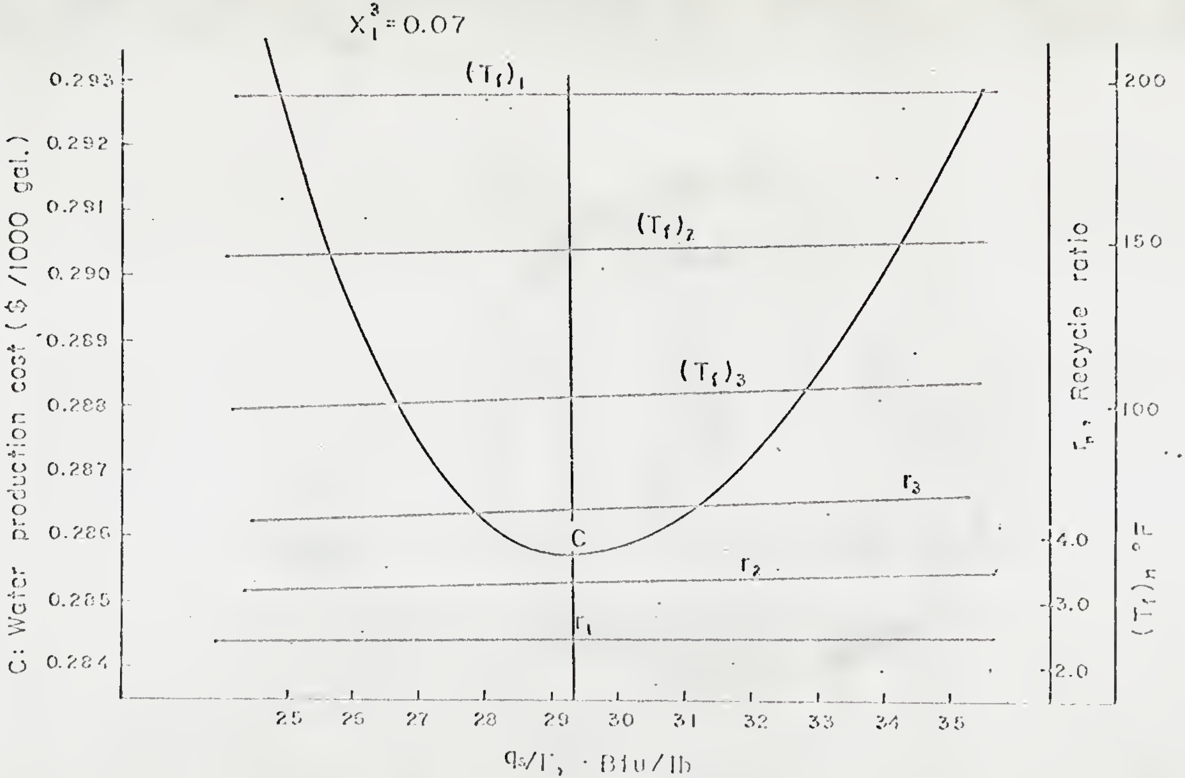


Fig.10. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 7$ % .

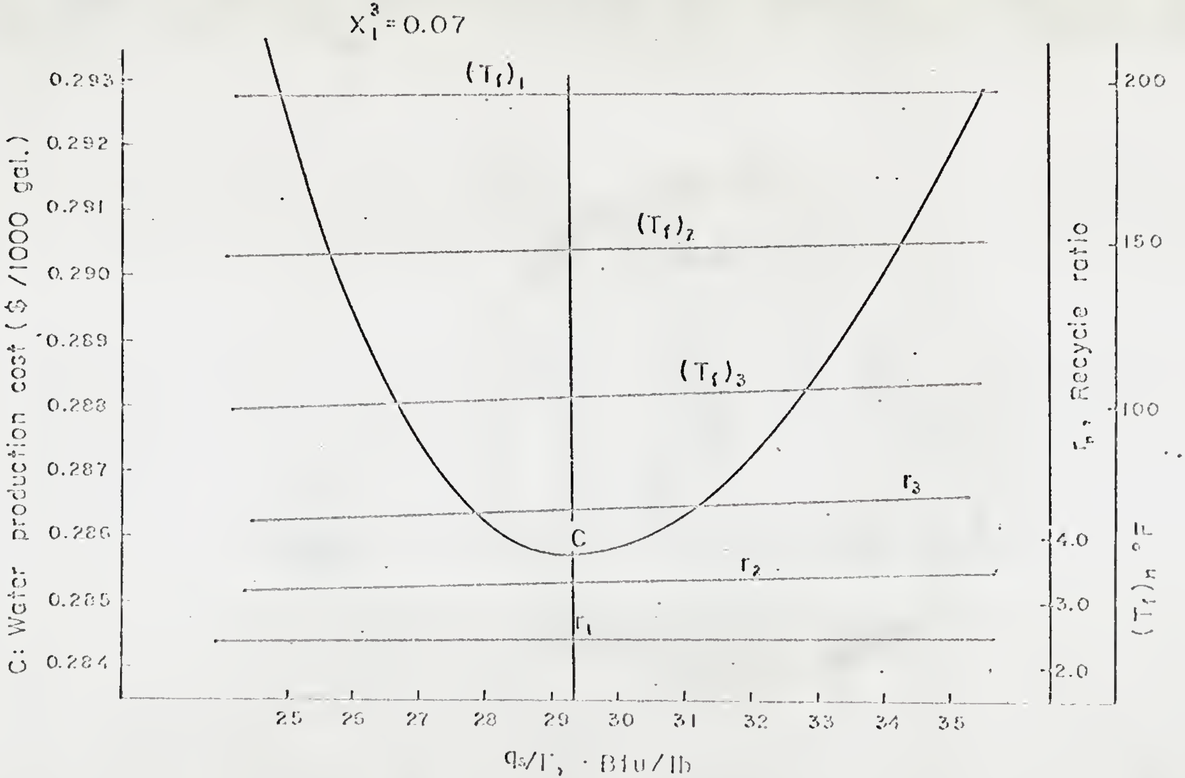


Fig.10. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 7\ \%$.

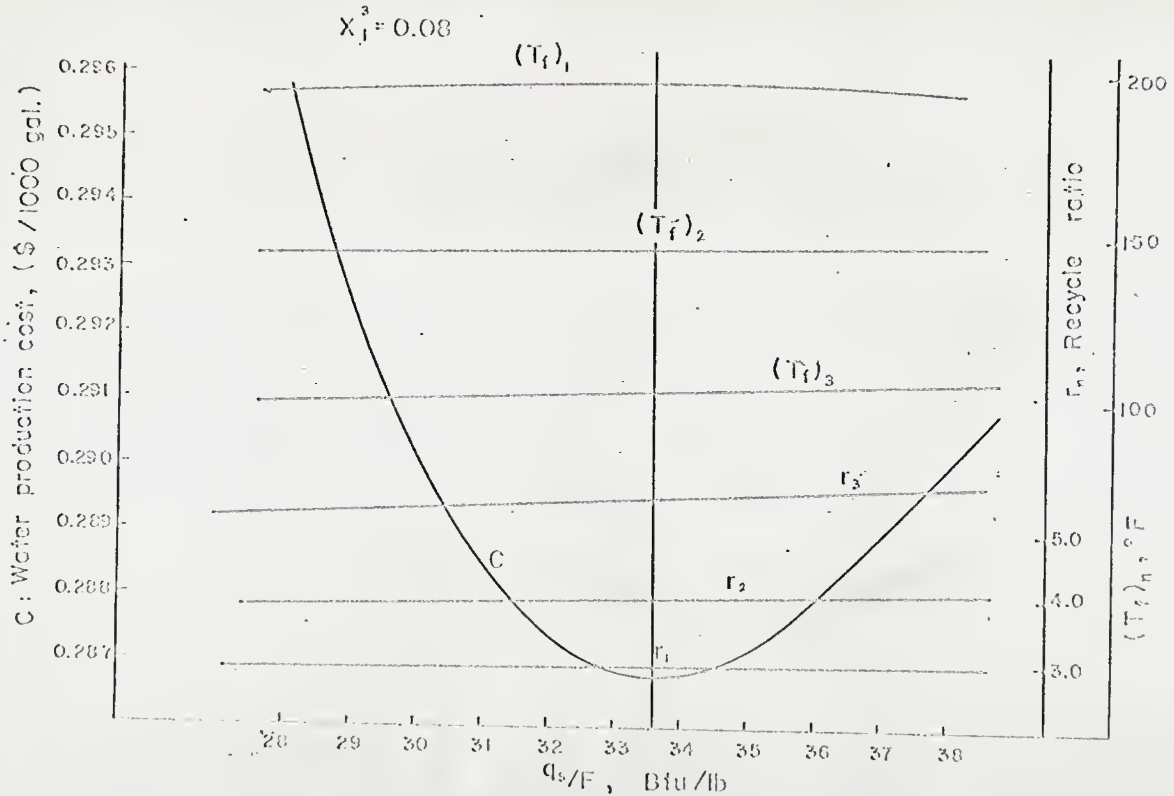
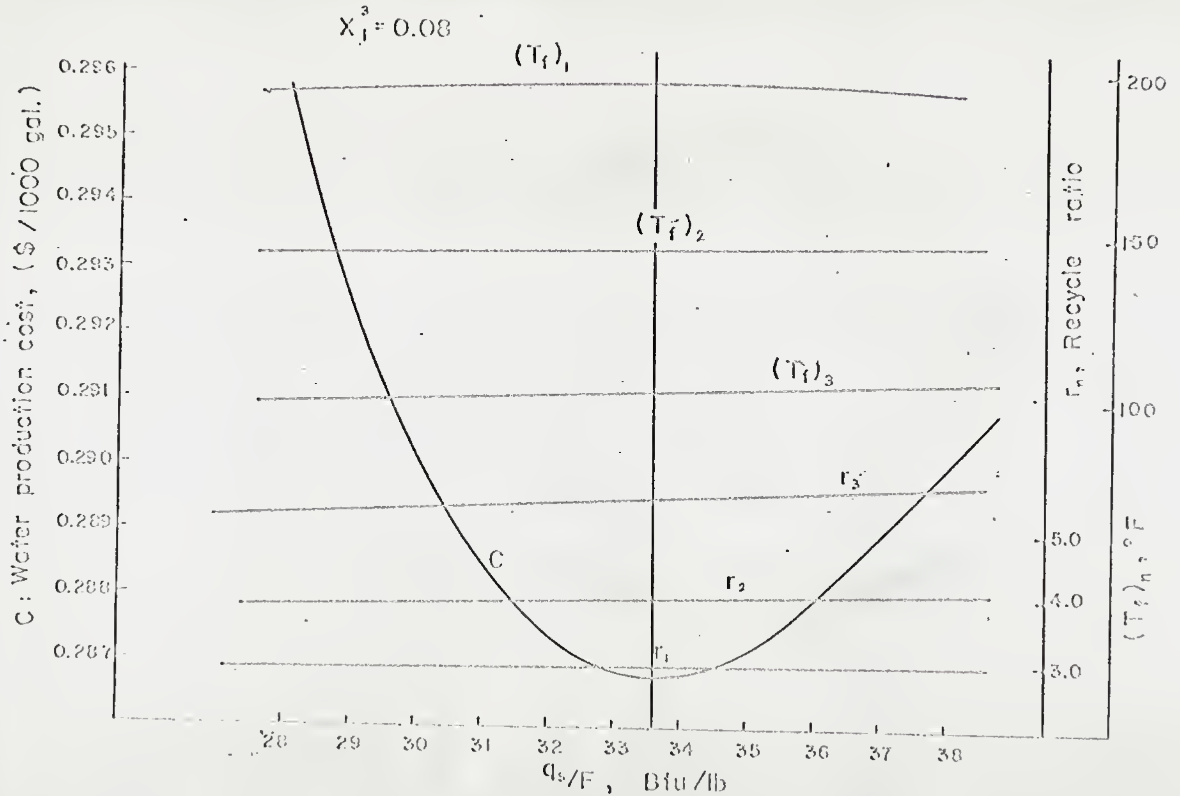


Fig. II. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 8$ % .



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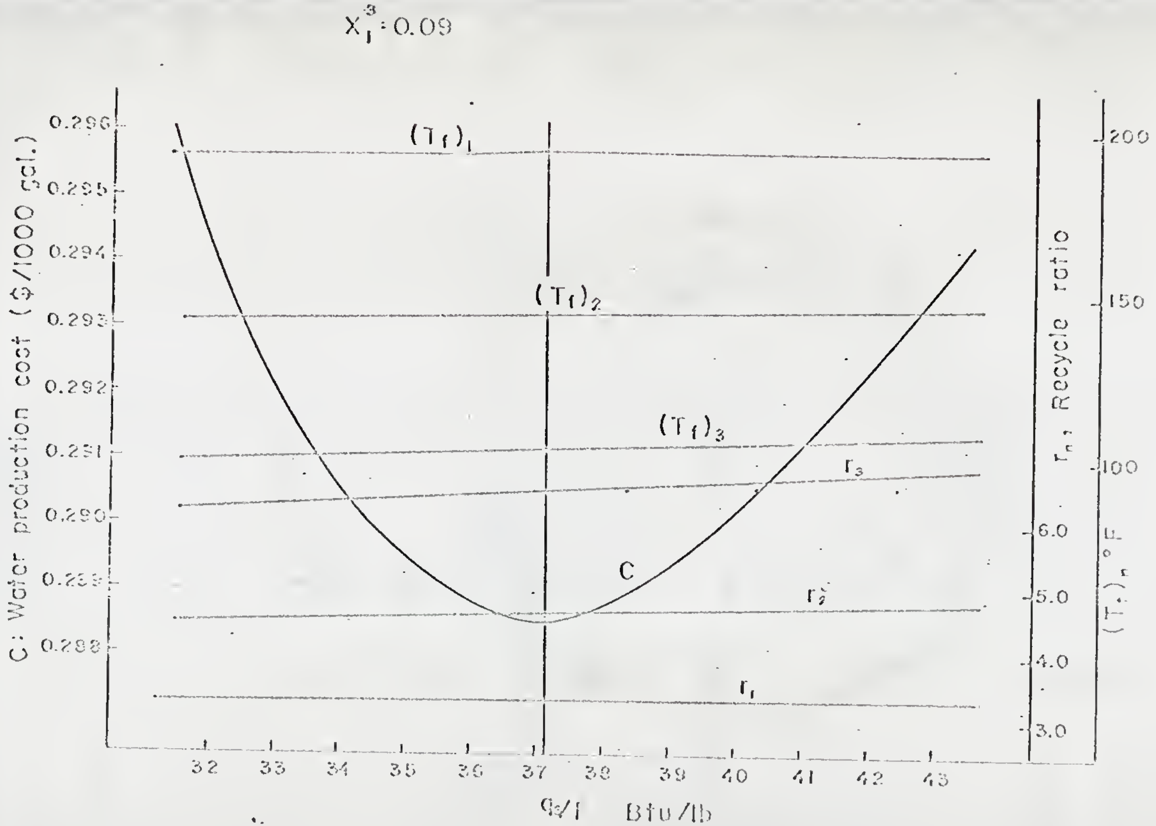

Fig.12. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 9\%$.

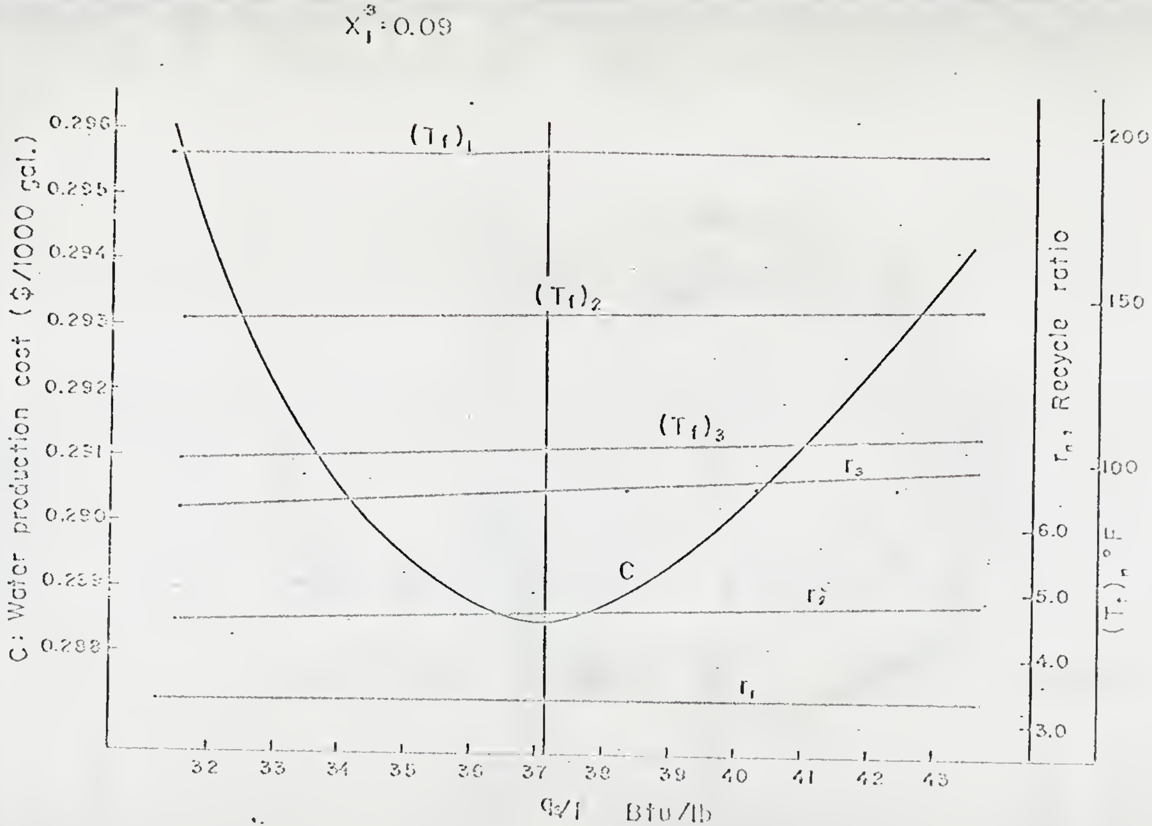

Fig.12. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 9\%$.

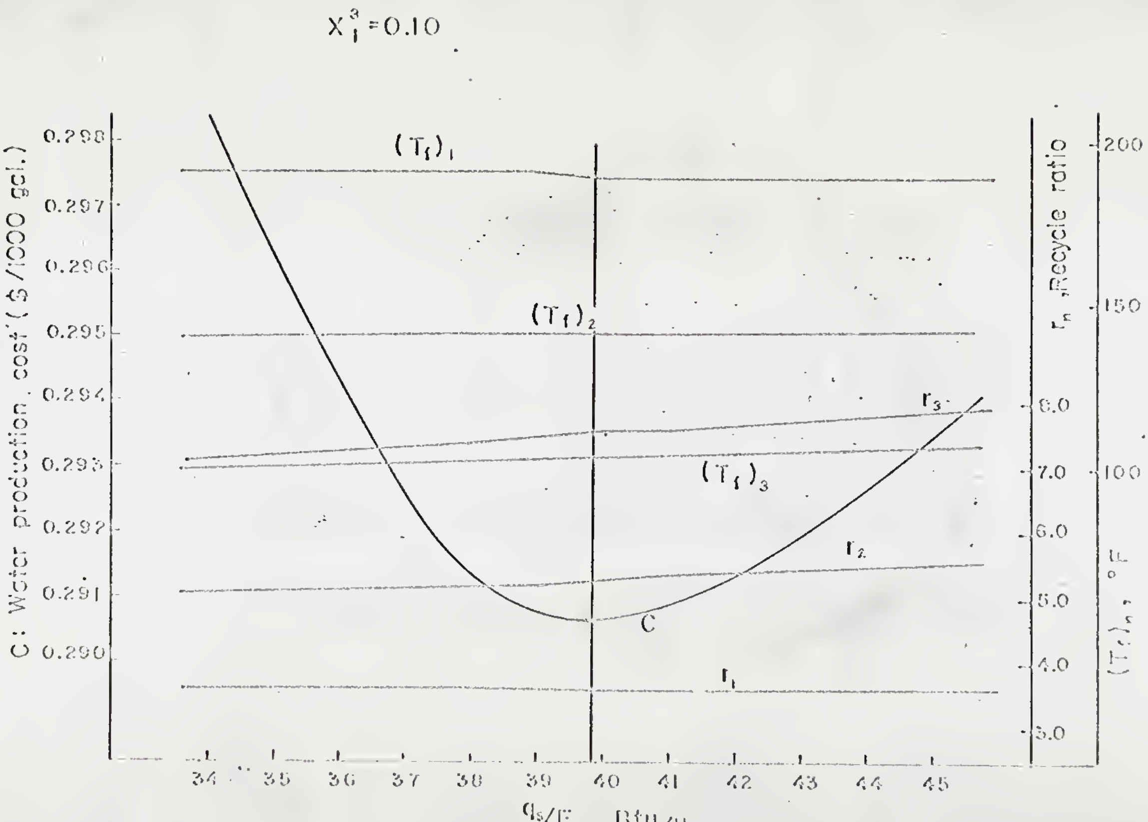

Fig.13. The cost and the optimization policies for sub-optimization problems with $x_1^3 = 10$ %.

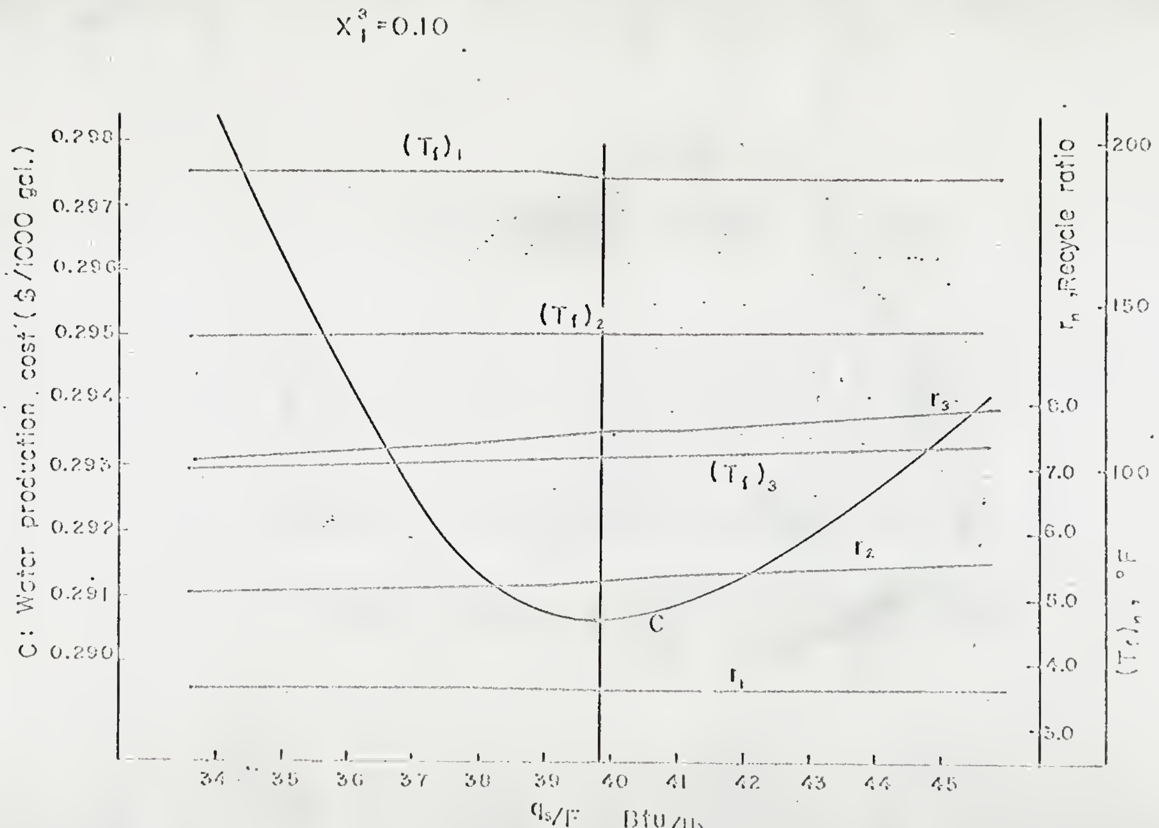

Fig.13. The cost and the optimization policies for sub-optimization problems with $x_1^3 = 10$ %.

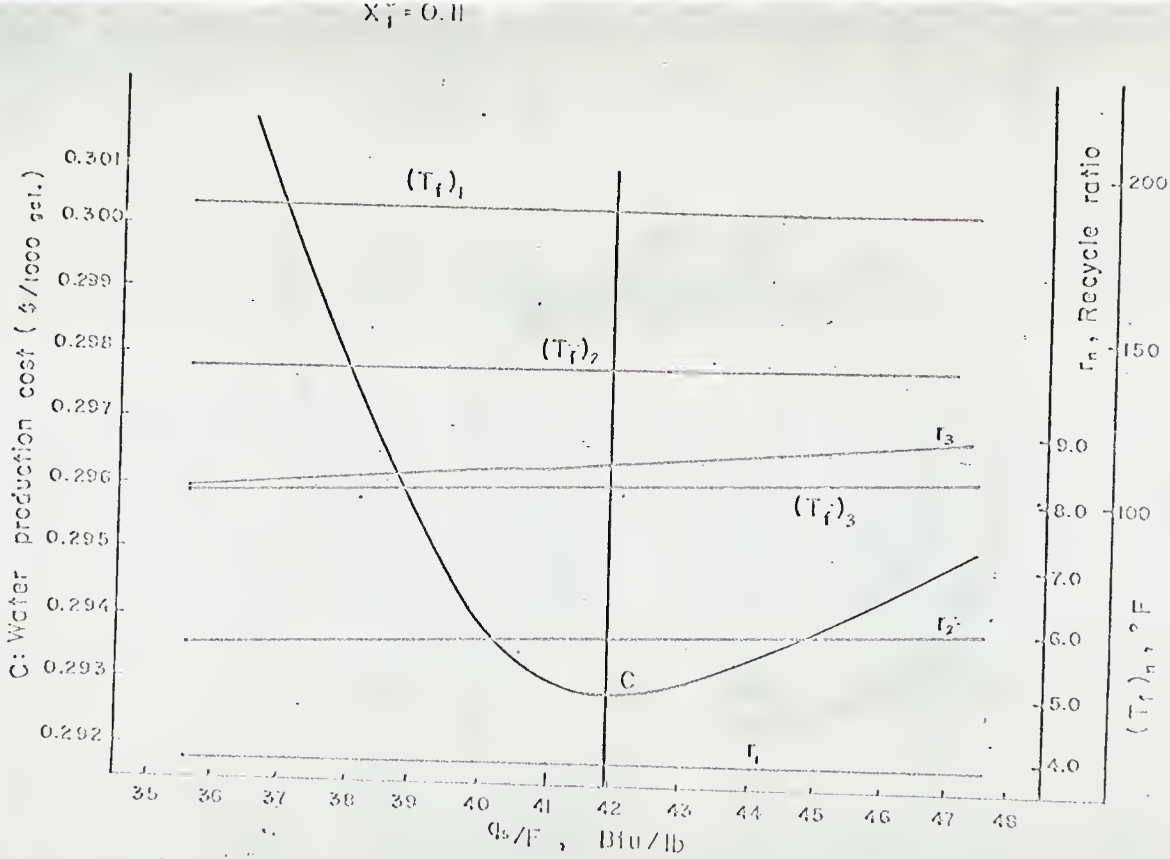

Fig.14. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 11$ % .

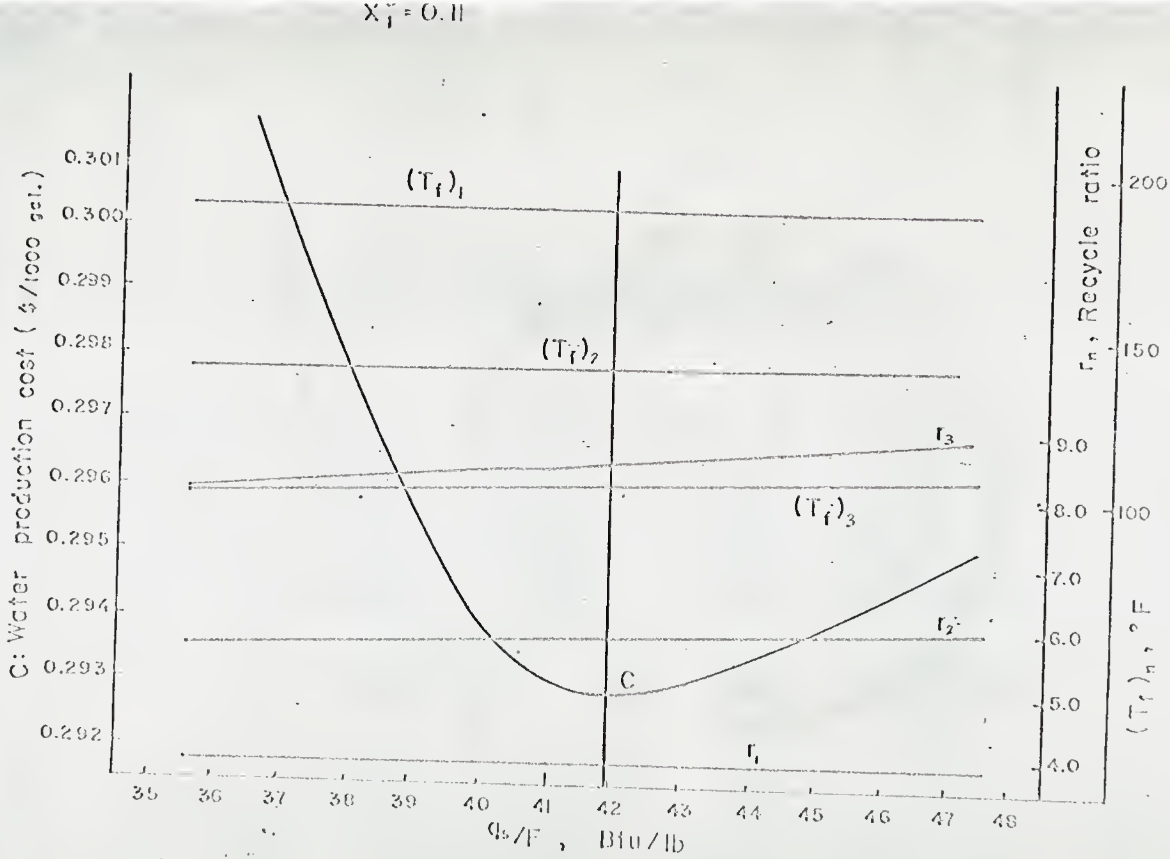

Fig.14. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 11$ % .

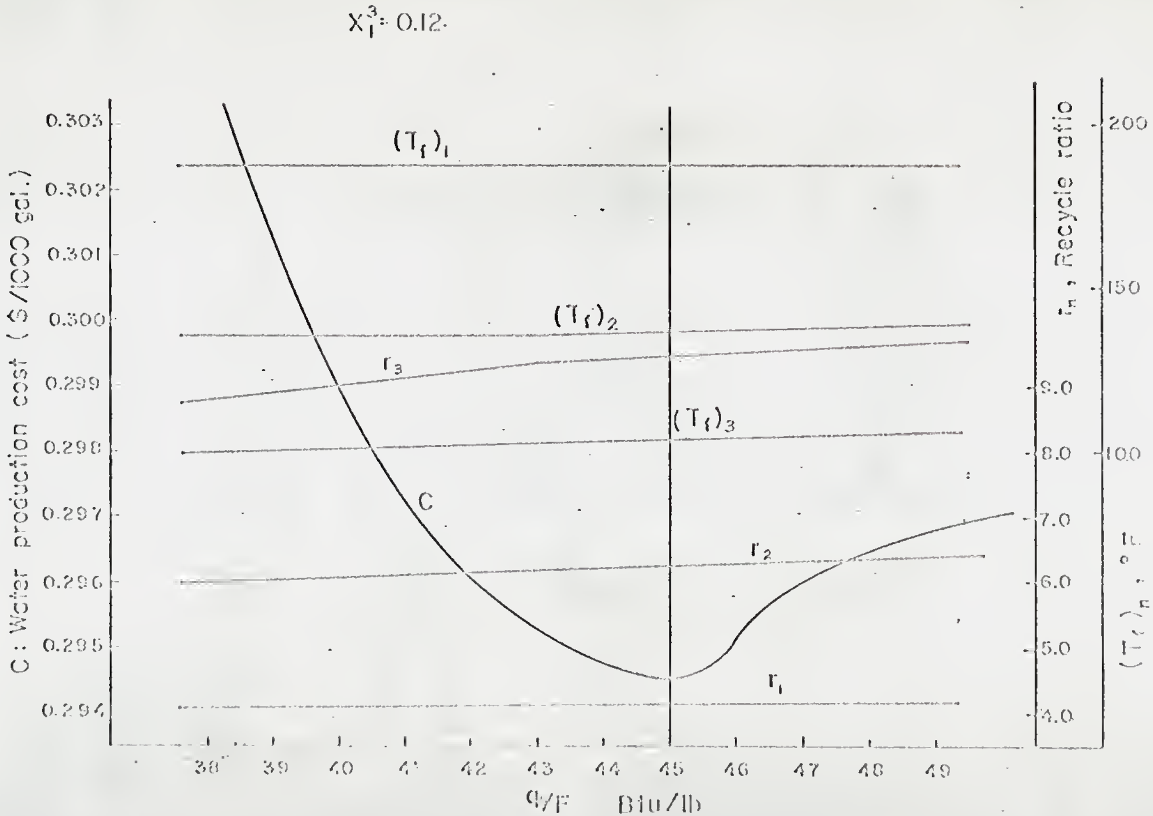

Fig.15. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 12$ %.

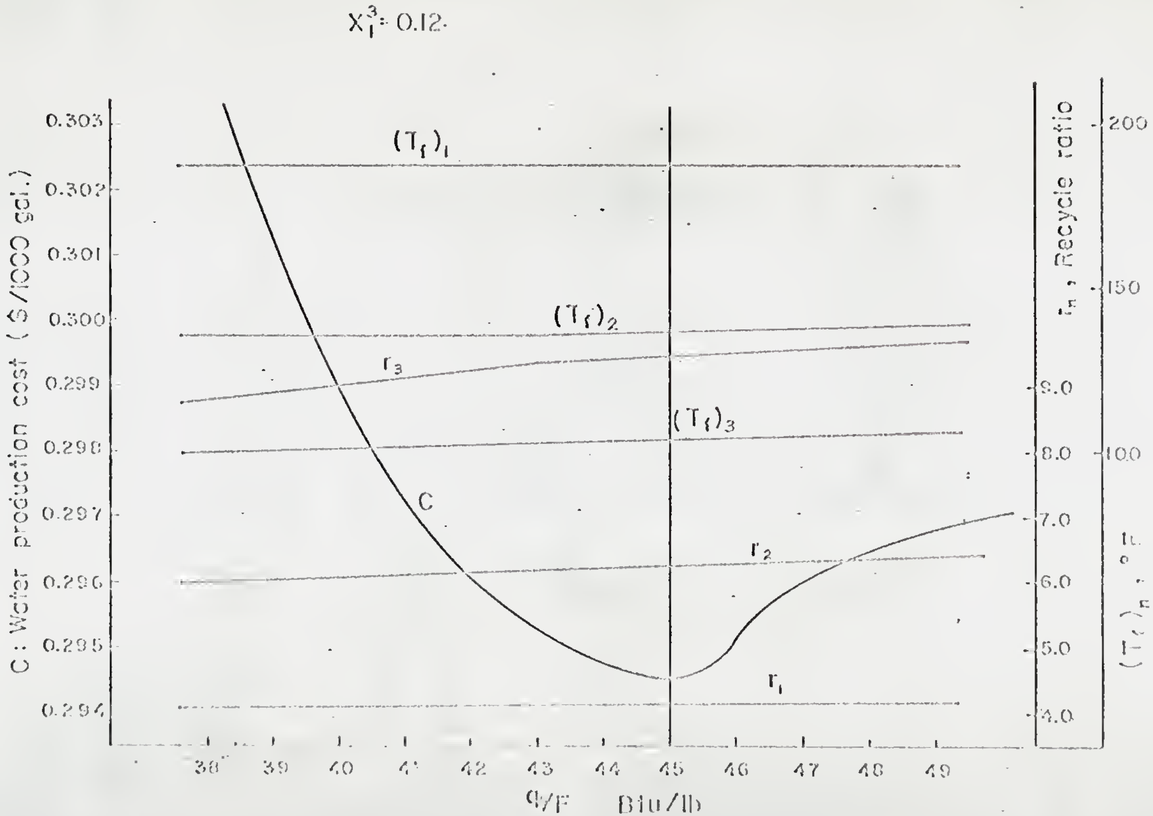

Fig.15. The cost and the optimum policies for sub-optimization problems with $x_1^3 = 12$ % .

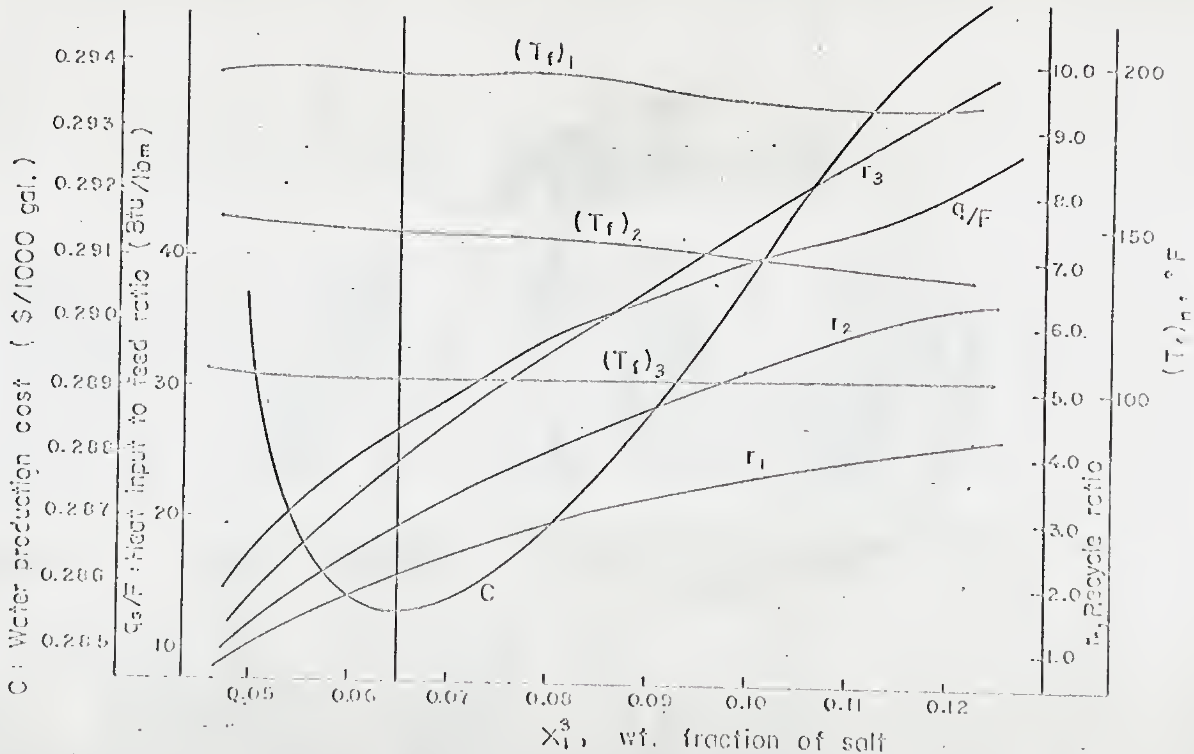

Fig.16. The cost and the optimum policies for the overall optimization problem.

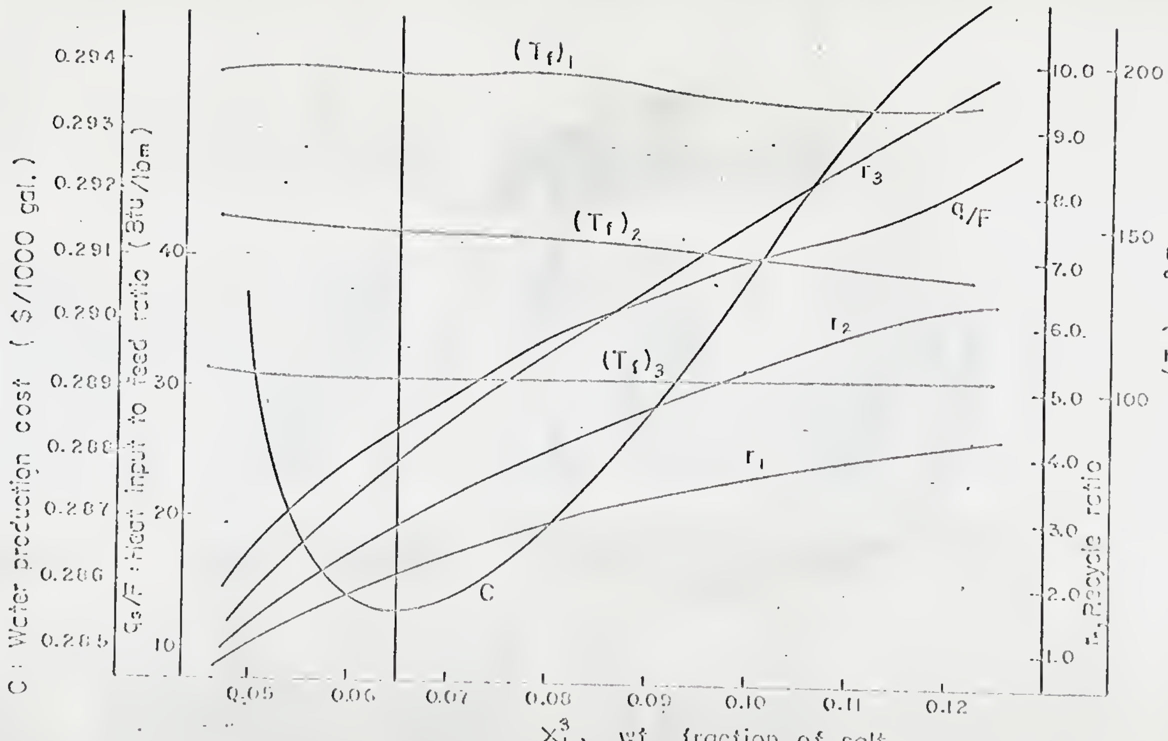

Fig.16. The cost and the optimum policies for the overall optimization problem.

temperature of the flashing brine leaving the first effect, $(T_f)_1 = 196^{\circ}F$,

temperature of the flashing brine leaving the second effect, $(T_f)_2 = 148^{\circ}F$,

temperature of the flashing brine leaving the third effect, $(T_f)_3 = 103^{\circ}F$.

5-4. Overall Optimum by the Simplex Method

It has been shown that the discrete version of the maximum principle can be applied to find the optimal condition for a sub-optimization problem with a set of given values for x_1^3 and q_s/F. If a multi-dimensional search technique is combined with the sub-optimization procedure using the maximum principle to minimize the objective function depending on the two variables, x_1^3 and q_s/F, which are fixed in each sub-optimization step, the overall optimal policy of the system can be obtained in a straight-forward manner by using one complete computer program.

A number of multi-dimensional search techniques are available, such as Powell's method (13), Box's method (16), Smith's method (15), etc. Powell's method is known to be an efficient method for finding the minimum of an objective function or simply a function without calculating its derivatives. However, Nelder and Mead (14) have recently developed the so-called "simplex method" which is reported to perform more efficiently than Powell's method. It is said that, for two-dimensional search problems, the efficiency of the simplex method far exceeds that of Powell's method. The simplex method was used in this study.

The general concept of this method for the minimization of a function of n variables is to set up a simplex of (n+1) vertices, that is, to select (n+1)

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points in the space of n variables and calculate values of the function at the selected points. Then, by comparing the calculated values of the function among themselves, the vertex with the highest value (i.e. the worst point in minimization) is replaced by another point with a lower value of the function, which is determined according to certain operations to be described later. The simplex method forces the function to approach the minimum by, at each stage of operation, discarding the worst point of a simplex and adapting a better point to form a new simplex. This procedure is repeated until the minimum point is achieved.

For the problem in hand, the simplex can be represented by a triangle as shown in Fig. 17. P_1, P_2, and P_3 are the points in the two dimensional space of x_1^3 and q_s/F, which define the current "simplex."

We define

y_n = the value of the objective function or the water cost x_2^3 at the point, P_n,

P_1 = the vertex with the lowest value of the objective function (y_1) in the simplex,

P_3 = the vertex with the highest value of the objective function (y_3) in the simplex,

P_2 = the vertex at which the corresponding value of the objective function (y_2) lies between (y_1) and (y_3),

P_4 = the centroid of the vertices, P_1 and P_2, with the value of the objective function (y_4).

The operations, through which a new point with a lower value of the objective function is found, are reflection, expansion and contraction. The reflection of the worst point, P_3, with respect to centroid, P_4, is denoted by P_5 and its co-ordinates are defined by the relation

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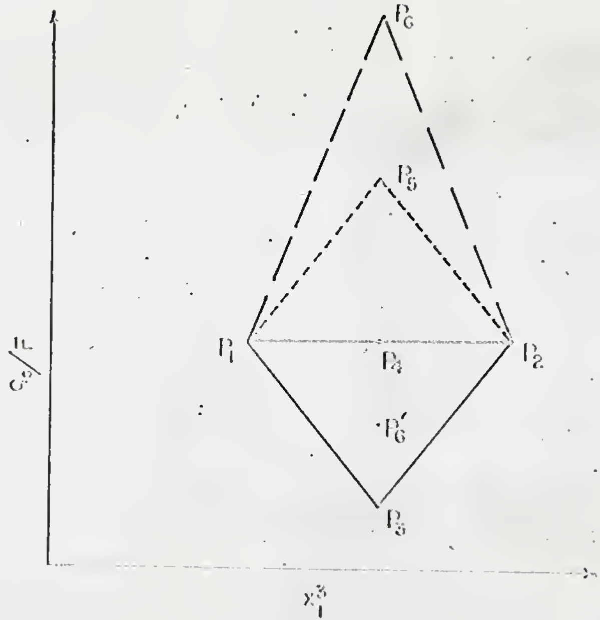

reflection : $P_5 = P_4 + \alpha'(P_4 - P_3)$

expansion : $P_6 = P_4 + \gamma'(P_5 - P_4)$

contraction : $P_6' = P_4 + \beta'(P_3 - P_4)$

$\alpha' = 1 \quad , \quad \beta' = \frac{1}{2} \quad , \quad \gamma' = 2$

Fig. 17 . Simplex triangle .

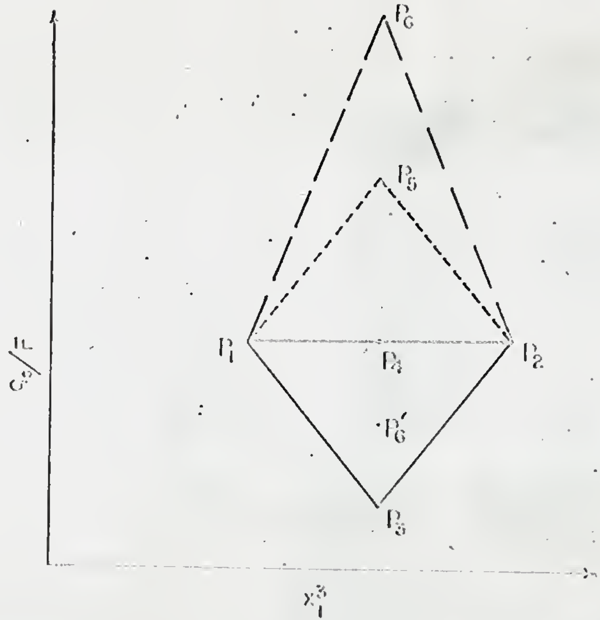

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Fig. 17 . Simplex triangle .

$$P_5 = P_4 + \alpha'(P_4 - P_3) \tag{104}$$

where α' is a positive constant, the reflection coefficient. Thus P_5 is on the line joining P_3 and P_4, on the far side of P_4 from P_3 with $\overline{P_5P_4} = \alpha'\overline{P_3P_4}$.

The reflected point P_5 may be expanded to P_6 by the relation

$$P_6 = P_4 + \gamma'(P_5 - P_4) \tag{105}$$

The expansion coefficient γ', which is greater than unity, is the ratio of the distance $\overline{P_6P_4}$ to $\overline{P_5P_4}$.

The contraction of the worst point, P_3, with respect to the centroid, P_4, is represented by P_6' and defined by the relation

$$P_6' = P_4 + \beta'(P_3 - P_4) \tag{106}$$

where β' is a positive number between 0 and 1 and is the ratio of the distance $\overline{P_6'P_4}$ to $\overline{P_3P_4}$. The values of these coefficients considered best by Nelder and Mead (14) are

$$\alpha' = 1, \qquad \beta' = \frac{1}{2}, \qquad \text{and} \quad \gamma' = 2.$$

The details of the procedure for using the method are described as follows:

First, P_3 is reflected to P_5, and if y_5 lies between y_1 and y_3, then P_3 is replaced by P_5 and we start the procedure again with a new simplex.

If $y_5 < y_1$, that is, if the reflection has produced a new minimum, then we expand P_5 to P_6. If $y_6 < y_1$, we replace P_3 by P_6 and restart the process. But if $y_6 > y_1$, then we have a failed expansion, and we replace P_3 by P_5 before restarting.

$$P_5 = P_4 + \alpha'(P_4 - P_3) \tag{104}$$

where α' is a positive constant, the reflection coefficient. Thus P_5 is on the line joining P_3 and P_4, on the far side of P_4 from P_3 with $\overline{P_5P_4} = \alpha'\overline{P_3P_4}$.

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If, after reflection, we find that $y_5 > y_1$ and $y_5 > y_2$, then we define a new P_3 to be either the old P_3 or P_5, depending on whichever has the lower y_n value, and then contract P_3 to P_6'. We then accept P_6' for P_3 and restart the procedure, unless $y_6' > y_3$, that is, the contracted point is worse than P_3. For such a failed contraction, we replace P_2 and P_3 by $\frac{(P_2 + P_1)}{2}$ and $\frac{(P_3 + P_1)}{2}$ respectively and restart the process.

A flow diagram of the method is given in Fig. 18, and a complete computer program for the simplex method together with the sub-optimization program by means of the discrete form of the maximum principle is given in Table A3 of the Appendix.

The optimal water production cost obtained by using this method is \$0.2855/1000 gal. and the corresponding optimal operating conditions are as follows:

salt concentration of the flashing brine leaving the third effect,	$x_1^3 = 0.065$,
the ratio of heat load to seawater feed	$q_s/F = 27$,
recycle ratio in the first effect,	$r_1 = 2.139$,
recycle ratio in the second effect	$r_2 = 2.877$,
recycle ratio in the third effect	$r_3 = 3.861$,
temperature of the flashing brine leaving the first effect,	$(T_f)_1 = 195.9^\circ F$,
temperature of the flashing brine leaving the second effect,	$(T_f)_2 = 147.7^\circ F$,
temperature of the flashing brine leaving the third effect,	$(T_f)_3 = 103^\circ F$.

The same procedures used here for the two dimensional search can be extended to the n-dimensional problem (14). The worst point of a simplex with (n+1)

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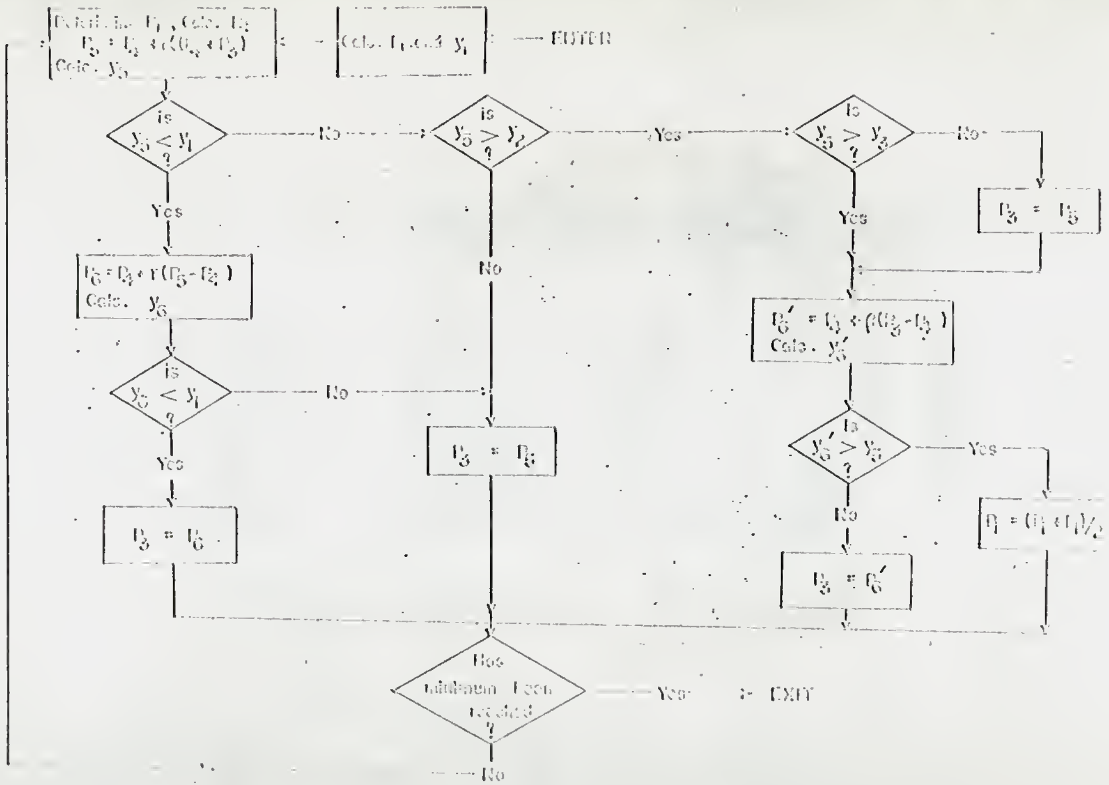

Fig. 18. Flow diagram for the simplex method.

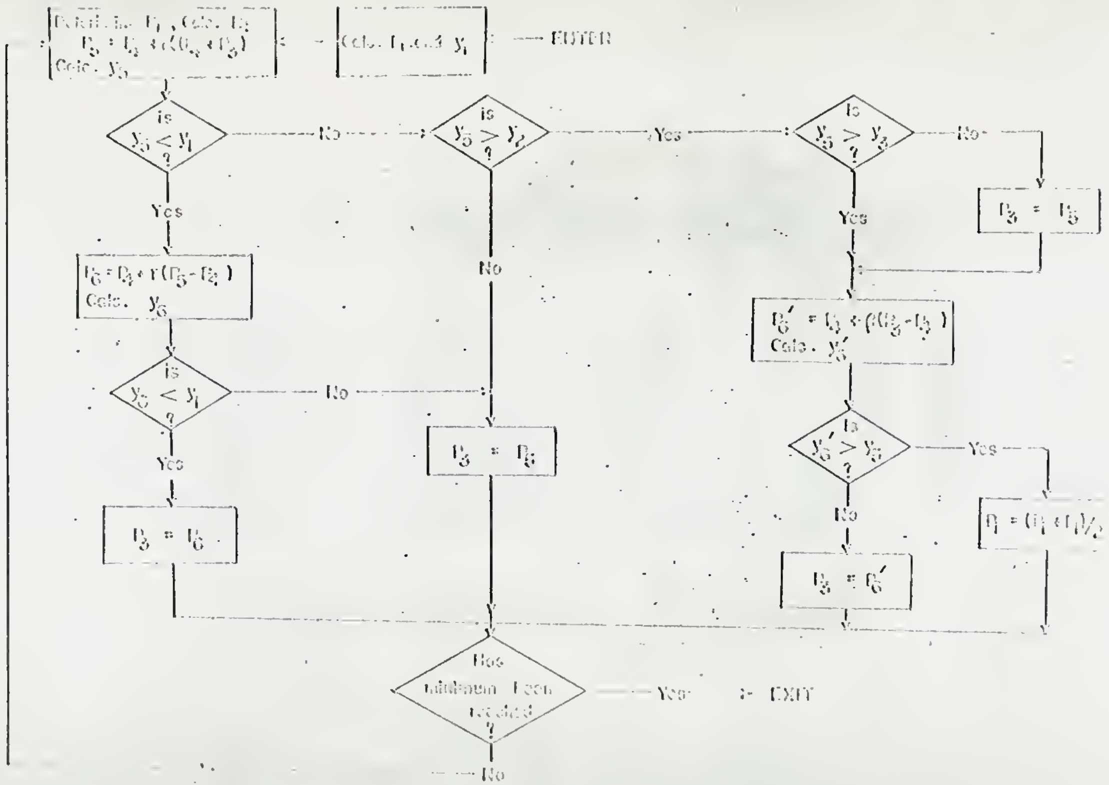

Fig. 18. Flow diagram for the simplex method.

vertices is reflected, expanded or contracted in the same manner with respect to the centroid of the remaining n vertices until the minimum point is attained.

5-5. Comparison of Results from the Use of the Two Search Techniques

The optimal policies from the two search techniques are tabulated in Table 4. Both in the sub-optimization stage and in the search stage, a criterion is adopted to test if the minimum point of the objective function is attained and no further iteration is needed. Theoretically when the objective functions attain their minimum points, it is necessary that the velues of the derivatives of the corresponding Hamiltonian functions, $\frac{\partial H}{\partial \theta}$, in the sub-optimization step and the "standard error" defined by

$$\sqrt{\sum_{i=1}^{3} (y_i - y_4)^2 / 3}$$

where y_i, i = 1, 2, 3, and y_4 are the values of the objective function at the vertices and centroid of the simplex respectively, in the search step are both equal to zero. In the actual calculation, these values are compared to some pre-set values or criteria and the iteration stops when they fall below such criteria. In the computer code developed, the criterion for $\frac{\partial H}{\partial \theta}$ is designated by ER and that for the "standard error" by ERROR.

The numerical results in column (a) of Table 4 from the parametric search are obtained by setting ER = 1×10^{-4}; in column (b) from the simplex method by setting ER = 1×10^{-4} and ERROR = 1×10^{-4}; in column (c) from the simplex method by setting ER = 0.5×10^{-4} and ERROR = 0.5×10^{-4}.

From the closeness of the numerical values of the various operating variables between columns (a) and (b), it can be concluded that the parametric search and the simplex method both lead to the same optimum point if the same criterion is used. However, for the parametric search, tedious exhaustive numerical and/or graphical search procedures must be carried out. But by using

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$$\sqrt{\sum_{i=1}^{3} (y_i - y_4)^2 / 3}$$

where y_i, i = 1, 2, 3, and y_4 are the values of the objective function at the vertices and centroid of the simplex respectively, in the search step are both equal to zero. In the actual calculation, these values are compared to some pre-set values or criteria and the iteration stops when they fall below such criteria. In the computer code developed, the criterion for $\frac{\partial H}{\partial \theta}$ is designated by ER and that for the "standard error" by ERROR.

The numerical results in column (a) of Table 4 from the parametric search are obtained by setting ER = 1×10^{-4}; in column (b) from the simplex method by setting ER = 1×10^{-4} and ERROR = 1×10^{-4}; in column (c) from the simplex method by setting ER = 0.5×10^{-4} and ERROR = 0.5×10^{-4}.

From the closeness of the numerical values of the various operating variables between columns (a) and (b), it can be concluded that the parametric search and the simplex method both lead to the same optimum point if the same criterion is used. However, for the parametric search, tedious exhaustive numerical and/or graphical search procedures must be carried out. But by using

Table 4. Optimal Policy, Operating Conditions, and Cost from the Two Search Techniques

Item	Symbol	Unit	Parametric Search (a)	Simplex Method (b)	Simplex Method (c)
Maximum allowable error	ER		1×10^{-4}	1×10^{-4}	0.5×10^{-4}
	ERROR		----	1×10^{-4}	0.5×10^{-4}
exit brine conc. of the 3rd effect	x_1^3	wt.frac.	0.065	0.065	0.06475
ratio of heat load to seawater feed	q_s/F	Btu/lb	27.0	27.0	26.64
recycle ratio in the 1st effect	r_1		2.14	2.139	2.125
recycle ratio in the 2nd effect	r_2		2.88	2.877	2.841
recycle ratio in the 3rd effect	r_3		3.86	3.861	3.819
exit brine temp. of the 1st effect	$(T_f)_1$	^oF	196.0	195.9	195.85
exit brine temp. of the 2nd effect	$(T_f)_2$	^oF	148.0	147.7	147.72
exit brine temp. of the 3rd effect	$(T_f)_3$	^oF	103.0	103.0	103.0
water production cost	x_2^3	\$/1000 gal.	0.2855	0.2855	0.2855

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Item	Symbol	Unit	Parametric Search (a)	Simplex Method (b)	Simplex Method (c)
Maximum allowable error	ER		$1x10^{-4}$	$1x10^{-4}$	$0.5x10^{-4}$
	ERROR		----	$1x10^{-4}$	$0.5x10^{-4}$
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water production cost	x_2^3	$/1000 gal.	0.2855	0.2855	0.2855

the simplex method, the same results are obtained in a straight forward manner without employing such tedious procedures. For example, if criteria for stopping the iteration are changed from one set of values to another, the only thing that should be done for the simplex method is just to change the input data for the criteria to the corresponding new value. In contrast to this, the parametric search requires rather laborious procedures, such as the preparation of a new set of nine figures similar to Figs. 8 through 16.

However, the great advantage of the parametric search lies in the fact that it gives insight into the MEMS system and gives detailed information about the influences of the variables under search, namely x_1^3 and q_s/F, on the water production cost and the other optimal policies. From Figs. 8 through 15, it is seen that both the values of the recycle ratio r_n and the brine temperature $(T_f)_n$ vary linearly and slightly with the value of q_s/F. But the water production cost changes considerably with the value of q_s/F. From Fig. 16, it is seen again that the values of $(T_f)_n$ are nearly constant; however, they vary slightly and linearly with the value of x_1^3. On the other hand, the optimal recycle ratio r_n and the optimal ratio of the heat load to seawater feed q_s/F and the water production cost vary greatly with the value of x_1^3.

The results can then be summarized as follows:

(1) The optimum temperature of the flashing brine leaving each effect, $(T_f)_n$, varies only slightly with the values of q_s/F and x_1^3.

(2) The optimal recycle ratio in each effect, r_n, depends on x_1^3 but varies only slightly with q_s/F.

(3) The optimal ratio of the heat load to seawater feed, q_s/F, varies significantly with x_1^3.

(4) The water production cost varies greatly with the values of x_1^3 and q_s/F.

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CHAPTER 6

CONCLUSION

In this study a detailed analysis of a MEMS process has been made and quantitative solutions between the operating variables have been obtained. The operating variables of the system are the heat input to the brine heater, the recycle ratio in each effect, the brine concentration and temperature leaving each effect, and the number of stages in each effect.

A mathematical model of the MEMS system containing these operating variables have been developed. The quantitative relations of the model, which contain the operating variables, are used to set up cost equations which relate the performance of the system to the unit cost of product water. These equations are then employed in the optimization study by means of the discrete maximum principle. The parametric search techniques and simplex method are used in conjunction with the maximum principle to find the overall optimal condition.

While the simplex method gives rise directly to the optimum point, the parametric search gives detailed information about the influences of the individual parameters on the water cost and the other operating variables. It is obvious from Fig. 16 that the brine temperature changes little as we change x_1^3. On the other hand the recycle ratio r_n and q_s/F do change considerably.

The overall optimal operating conditions are summarized in Table 5, and the corresponding capital and operating costs, the various cost items, and their contributions to the overall fresh water cost on a percentage basis are given in Table 6.

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The overall optimal operating conditions are summarized in Table 5, and the corresponding capital and operating costs, the various cost items, and their contributions to the overall fresh water cost on a percentage basis are given in Table 6.

Table 5. The Overall Optimal Operating Conditions

Symbols	Items	Numerical values
(1) Concentrations $(C_f)_n$		
C_F	concentration of sea water feed	0.035 wt. fraction
$(C_f)_1'$	brine conc. entering the 1st effect	0.0397 wt. fraction
$(C_f)_1$	brine conc. leaving the 1st effect	0.0419 wt. fraction
$(C_f)_2'$	brine conc. entering the 2nd effect	0.0488 wt. fraction
$(C_f)_2$	brine conc. leaving the 2nd effect	0.0512 wt. fraction
$(C_f)_3'$	brine conc. entering the 3rd effect	0.0622 wt. fraction
$(C_f)_3$	brine conc. leaving the 3rd effect	0.065 wt. fraction
(2) Temperature $(T_f)_n$		
$(T_f)_1$	brine temp. leaving the 1st effect	$196^{\circ}F$
$(T_f)_2$	brine temp. leaving the 2nd effect	$148^{\circ}F$
$(T_f)_3$	brine temp. leaving the 3rd effect	$103^{\circ}F$
$(T_f)_0$	brine temp. entering the 1st effect	$250^{\circ}F$
$(T_j)_3$	sea water temperature	$85^{\circ}F$
(3) Recycle Ratios r_n		
r_1	recycle ratio in the 1st effect	2.14
r_2	recycle ratio in the 2nd effect	2.88
r_3	recycle ratio in the 3rd effect	3.86

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Table 5. The Overall Optimal Operating Conditions (Continued)

Symbols	Items	Numerical Values
(4) Flow Rates of Various Streams*		
F	flow rate of sea water feed	2170 gal./hr.
$(L)_{1'}$	brine stream entering the 1st effect	6810 gal./hr.
$(L)_1$	brine stream leaving the 1st effect	1810 gal./hr.
R_1	recycle flow rate in the 1st effect	4640 gal./hr.
$(L)_{2'}$	brine stream entering the 2nd effect	7025 gal./hr.
$(L)_2$	brine stream leaving the 2nd effect	1485 gal./hr.
R_2	recycle flow rate in the 2nd effect	5215 gal./hr.
$(L)_{3'}$	brine stream entering the 3rd effect	7210 gal./hr.
$(L)_3$	brine stream leaving the 3rd effect	1170 gal./hr.
R_3	recycle flow rate in the 3rd effect	5725 gal./hr.
R_4	cooling water flow rate	1220 gal./hr.
W_1	water production in the 1st effect	360 gal./hr.
W_2	water production in the 2nd effect	325 gal./hr.
W_3	water production in the 3rd effect	315 gal./hr.
(5) Heat Loads in the Brine Heater*		
q_s	heat load in the brine heater	4.87×10^5 Btu/hr.
q_s/F	ratio of q_s and F	27
q_s/λ_s	steam consumption in the brine heater	524 lbs/hr.
$\Sigma W/(q_s/\lambda_s)$	lbs. of fresh water produced per lb. of steam consumed	16

*Remark: Basis, 1000 gallon/hr. of fresh water production

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Table 6. Capital and Operating Cost Allocation

Symbols	Items	cost ($/1000 gal)	Percentage(%)
(1)	Capital cost	0.10805	37.838
E_2	brine heater	0.00125	0.439
ΣE_3^n	heat transfer area	0.08470	29.661
ΣE_5^n	pump	0.00130	0.455
E_6	outshell	0.02080	7.283
(2)	Operating cost	0.17752	62.162
E_1	steam	0.13131	45.979
E_7	feed brine	0.03247	11.370
E_8	cooling water	0.00568	1.991
ΣE_4^n	pumping power	0.00806	2.822
(3)	Total water production cost	0.28557	100.000

Remarks:

1. Basis: 1000 gallon fresh water production per hour.
2. Feed brine cost: $0.015/1000 gal. of sea water.
3. Cooling water cost: $0.005/1000 gal. of sea water.

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If a more realistic water production cost is required, we merely have to incorporate the updated cost data and cost equation into the scheme and apply the optimization techniques developed in this study for the MEMS system. Obviously this optimization technique can be applied to the other sequential multistage and multidecision processes.

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NOMENCLATURE

a = steam temperature, °R

A = heat transfer area, ft^2

B' = coefficient in the Clausius-Clapeyron equation, lb_f/ft^2

$B = (1 + \eta_f) B'$

C_B = capital cost per unit of heat transfer area in the brine heater, $\$/ft^2$

C_c = unit cooling water cost, \$/lb

$C_{cd} = \psi C_H$, $\$/ft^2$

C_e = unit power cost, \$/hp

$(C_f)_n$ = salt concentration of flashing brine at location n, wt.%

C_H = capital cost per unit of heat transfer area in the condensing chamber, $\$/ft^2$

$C_{ht} = \psi C_B$, $\$/ft^2$

C_J = capital cost per horsepower for the recycle pumps, \$/hp

C_F = salt concentration in the seawater feed, wt.%

C_p = heat capacity of water per pound, Btu/lb,°F.

$C_{pp} = C_J + C_e/\eta_p$, \$/ft-lb

C_{st} = unit steam cost, \$/lb

P_c = unit pretreatment cost for seawater feed, \$/lb

E = various cost items

F = flow rate of seawater feed, lb/hr

H^n = the Hamiltonian function at stage n

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H^n = the Hamiltonian function at stage n

H-1, H-2, and H-3 = the heating sections in the first, second and third effects respectively

HR-1, HR-2 and HR-3 = the heat recovery sections in the first, second and third effects respectively

h_c = unit enthalpy per pound of condensate, Btu/lb

h_f = unit enthalpy per pound of flashing brine, Btu/lb

h_j = unit enthalpy per pound of non-flashing brine, Btu/lb

H_v = unit enthalpy per pound of water vapor, Btu/lb

J_1, J_2 and J_3 = the circulation pumps in the first, second and third effects respectively

L = flow rate of the flashing brine, lb/hr

M_1, M_2 and M_3 = the mixing points in the first, second and third effects respectively

N_1, N_2 and N_3 = the number of stages in the first, second and third effects respectively

P_1 = the vertex of a simplex with the lowest function value, y_1

P_2 = the vertex of a simplex with function value y_2, y_1 y_2 y_3

P_3 = the vertex of a simplex with function value y_3 or the worst point of the simplex

P_4 = the centroid between points P_2 and P_1

P_5 = the point obtained after reflection of P_3

P_6 = the point obtained after expansion of P_3

P_6' = the point obtained after contraction of P_3

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P^o = vapor pressure of pure water, lb_f/ft^2

$\bar{P}$ = vapor pressure of the aqueous solution, lb_f/ft^2

q = heat transfer rate, Btu/hr

q_s = heat input per unit time in the brine heater, Btu/hr

R = ideal gas constant, Btu/lb,°R

R_1, R_2 and R_3 = the flow rates of recycle brine stream in the first, second and third effects respectively

R-1, R-2 and R-3 = the heat rejection sections in the first, second and third effects respectively

R_4 = flow rate of cooling water, lb/hr

r_1, r_2 and r_3 = the recycle ratios in the first, second and third effects respectively

s = the objective function, $/1000gal.

T_c = condensing temperature, °R

$(T_f)_n$ = temperature of the flashing brine at location n, °R

T_j = temperature of the non-flashing brine, °R

T_s = steam temperature at the brine heater, °R

$(\Delta T) = T_f - T_j$, °F

$(\Delta t) = T_c - T_j$, °F

U = overall heat transfer coefficient, Btu/hr. ft^2 °F

V = vapor rate, lb/hr

W_1, W_2 and W_3 = the rates of condensate formation in the first, second and third effects respectively, lb/hr

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ΣW_n = total water production rate, lb/hr

$x_1^o = C_F$, wt.%

$x_1^n = (C_f)_n$, wt.%

x_2^n = accumulated water cost in the first n-th effects, ¢/1000gal.

$x_3^n = (T_f)_n$, °R

$x_3^o = (T_f)_0$, temperature of the flashing brine entering the first effect, °R

y_n = the function value or water cost at point P_n

z_i^n = the adjoint variable in association with state variable x_i^n

GREEK LETTERS

$\alpha = T_f - T_c$,°F

α_n = the average of α in the n-th effect

α' = reflection coefficient, 1

β = vapor pressure depression of brine streams, lb_f/ft^2

β' = contraction coefficient, 0.5

γ' = expansion coefficient, 2

η_f = fractional increase in pumping power required due to friction

η_p = pump efficiency

$\theta_1^n = r_n$, recycle ratio in the n-th effect

$\theta_2^n = (T_f)_n$, °R

$\theta_3^n = \theta_2^n - \theta_2^{n-1} = \theta_2^n - x_3^{n-1}$, °F

ΣW_n = total water production rate, lb/hr

$x_1^0 = C_F$, wt.%

$x_1^n = (C_f)_n$, wt.%

x_2^n = accumulated water cost in the first n-th effects, $/1000gal.

$x_3^n = (T_f)_n$, °R

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λ = latent heat of vaporization , Btu/lb

λ_s = latent heat of steam at 274.4°F and 45 psia, Btu/lb

ρ = density of water, lb/cu.ft.

ψ = capitalization charge

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PART TWO

ANALYSIS AND OPTIMIZATION OF THE REVERSE OSMOSIS DESALINATION PROCESS

PART TWO

ANALYSIS AND OPTIMIZATION OF THE REVERSE OSMOSIS DESALINATION PROCESS

CHAPTER 1

INTRODUCTION

The fugacity of the solvent in a solution is always lower than that of the pure solvent under the same pressure as the solution (17). If there is a semipermeable membrane present between the solution and pure solvent, which will pass solvent molecules preferentially or exclusively, there will be a net flow of pure solvent into the solution. At an equilibrium state the fugacities of the solvent in the solution and in the pure state become equal, and therefore there is no net solvent flow into the solution. This state of equilibrium can be brought about by raising the pressure of the solution to its osmotic pressure (18). If the pressure on the solution is raised above the equilibrium osmotic pressure then the pure solvent will flow out of the solution. This is the reverse of the osmotic process or the so-called reverse osmosis process. This method requires no phase change, therefore, it has an inherent advantage over distillation and freezing desalination processes from the point of view of energy requirement. However, progress on reverse osmosis depends greatly on the availability of a suitable semipermeable membrane which can stand up well over time to the required pressures and still give an appreciable flow of potable water. It was not until recently that such a synthetic membrane was developed and serious consideration was given to the use of reverse osmosis in desalination.

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Much attention has been drawn to the improvement of membrane fabrication techniques (19), but only little attention has been

directed to the system analysis of the process itself. Merten, et. al., has given an extensive cost analysis of a single stage system (20). Investigators at Kansas State University (21, 22, 23) have proposed a completely mixed model of the multi-stage sequential system based on the assumption of uniform salt concentration inside the osmosis unit, and the optimization of this model has been carried out by the same group (21, 22, 23). In the present study, the plug flow model is proposed by taking into account the concentration change inside the tubular osmosis unit. The proposed plug flow model of a multi-stage sequential system is described in Chapter 2. The quantitative relations between operating variables are derived in Chapter 3. Various cost functions of the system are established in Chapter 4. The outline of the optimization procedure of the system is described in Chapter 5.

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CHAPTER 2

PROCESS DESCRIPTION

A simplified flow diagram of a N-stage reverse osmosis system is shown in Fig. 1. The principal components of the system are listed in Fig. 1. Each stage consists of a membrane separator unit, MS; a high pressure pump, J_1; and a recirculation pump, J_2. q, R, and W represent respectively the flow rate of brine stream, recycle brine, and water produced. Superscript n is used to indicate the quantity referred to the n-th stage. However, the subscripts i and e refer respectively to the inlet and exit quantities to the membrane separator. In addition, the blowdown turbine at the end of the process is represented by J_3.

Sea water is first brought through a prefilter and is introduced into the first stage as the brine stream, q^o. It is then pumped by the high pressure pump, J_1^1, to an operating pressure in excess of its osmotic pressure and then mixed with recycle brine, R^1, at the mixing point, M^1. The resulting combined stream, q_1^1, is carried through the membrane separator unit, MS by means of the recirculation pump, J_2^1. The membrane separator is a shell-and-tube arrangement. The solvent water of the brine stream under a pressure higher than its osmotic pressure migrates across the semipermeable membrane tube to the lower pressure shell side of the separator. The collected water product from the shell side of the first stage, W^1, is then introduced and stored in the fresh-water reservoir.

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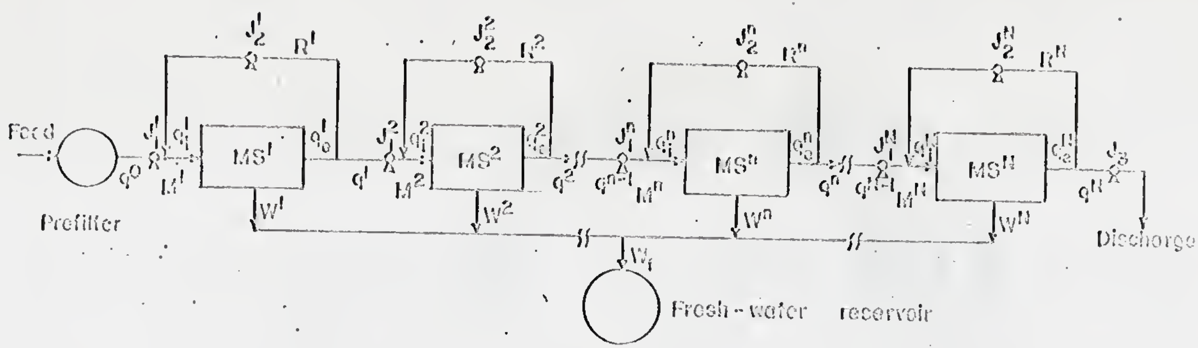
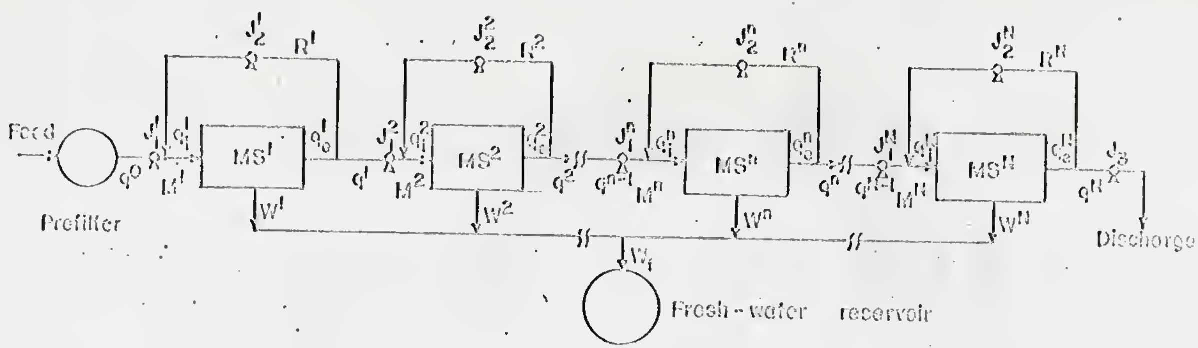


Fig. 1. Schematic diagram of a sequential N-stage reverse osmosis water purification process.

Where : J_1^n : the high pressure pump at n^{th} stage.

J_2^n : the recirculation pump at n^{th} stage.

J_3 : the blowdown turbine at the end of the process.



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and the other stream, R^1, is recirculated by the pump J_2^1, and then mixed with the sea-water feed at the mixing point M^1. The operation of the subsequent stages is similar to that of the first stage except at the last stage, where the brine stream, q^N, is allowed to blow down through a recovery turbine before it is rejected as waste.

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CHAPTER 3

PROCESS ANALYSIS

A fairly general model of a sequential reverse osmosis desalination system will be set up first. The first several sections are devoted to the derivation of the system equations of such a model. Three simplified versions of such a model are then proposed in the last section.

A schematic representation of this model is shown in Fig. 1 and the n-th stage of this model is depicted in Fig. 2. Each stage consists of a membrane separator unit, a recirculation pump and a high pressure pump. The last stage, however, includes, in addition, a blowdown turbine.

The flow rates of the brine stream, q, recycle brine, R, and water production, W, and the superscript and subscript representations have been defined in chapter 2. Definitions of several other symbols employed in the derivation are listed below:

x^n = the mass fraction of salt in the brine stream leaving the n-th stage,

x_i^n = the mass fraction of salt in the brine stream entering the membrane separator of the n-th stage,

x_e^n = the mass fraction of salt in the brine stream leaving the membrane separator of the n-th stage,

q^o = the mass flow rate of the sea water feed (lb_m/hr),

x^o = the mass fraction of salt in the sea water feed,

r^n = the recycle ratio in the n-th stage defined as the ratio of R^n and q^{n-1},

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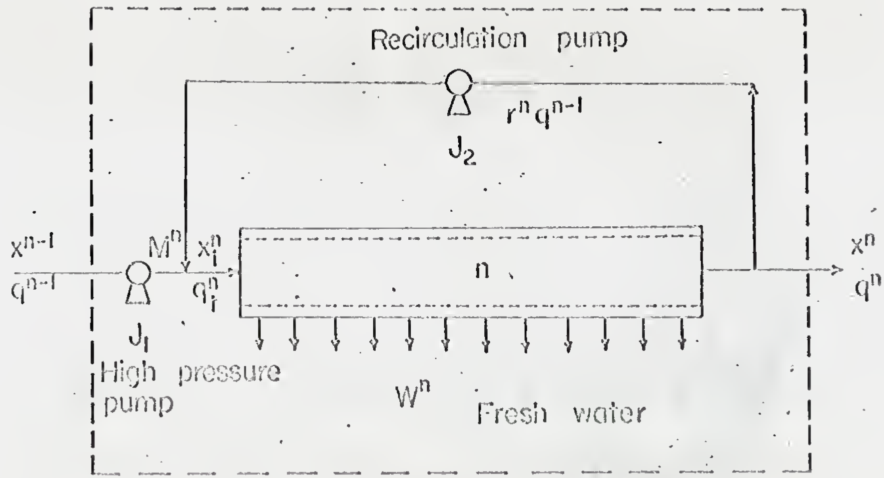

Fig. 2. The n-th stage of a general model.

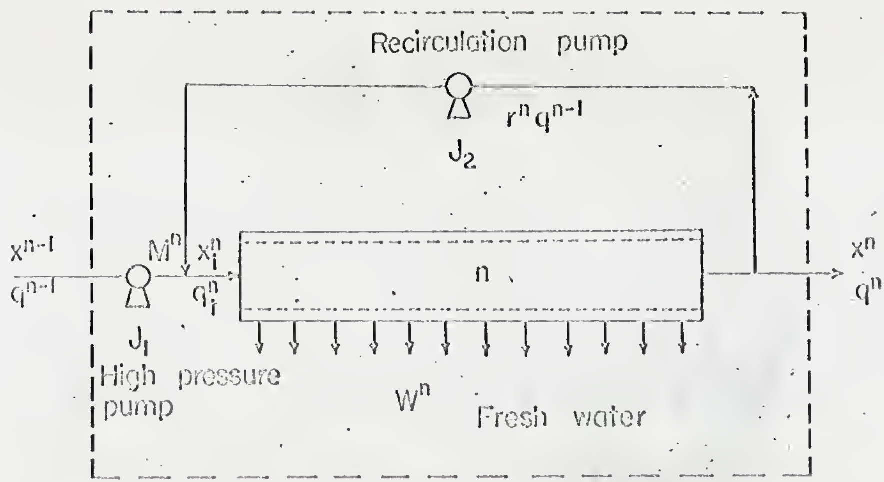

Fig. 2. The n-th stage of a general model.

N = the total number of stages in the sequence of the process,

W_f = the total mass flow rate of the fresh water produced from the whole system (lb_m/hr), i.e.

$$W_f = \sum_{n=1}^{N} W^n$$

p^n = operating pressure at the n-th stage (psi),

ΔP^n = the pressure difference across the membrane of the n-th stage (psi),

S^n = the membrane area of the n-th stage (ft^2).

3-1. The Fresh Water Production Rate W^n and W_f.

The material balance around the n-th stage is

$$q^{n-1} = W^n + q^n \qquad n = 1, 2, \ldots, N. \tag{1}$$

The material balance for the process as a whole is

$$q^0 = W_f + q^N \tag{2}$$

A salt material balance for the first n stages gives

$$q^n = \frac{q^0 x^0}{x^n} \qquad n = 1, 2, \ldots, N. \tag{3}$$

Substituting equations (3) into equation (1) yields

$$W^n = q^0 x^0 \left(\frac{1}{x^{n-1}} - \frac{1}{x^n}\right) \tag{4}$$

Substituting equation (3) into equation (2) yields

$$W_f = q^0 \left(1 - \frac{x^0}{x^N}\right) \tag{5}$$

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3-2. The Volumetric Flux of Water Through the Membrane, F

The volumetric flux of water, F, through a membrane of constant permeability has been reported (21, 22, 23) as

$$F = \frac{K(\Delta P - 12{,}000\ x)}{1 + 3.05 \times 10^5 \dfrac{K\ d}{(Sc)^{1/3} D_a} \dfrac{x}{Re^{7/8}}} \tag{6}$$

where

F = water flux, $\left(\frac{ft^3}{ft^2\text{-}hr}\right)$,

K = the membrane constant, $\left(\frac{ft^3}{ft^2\text{-}hr\text{-}psi}\right)$,

ΔP = the pressure difference across the membrane (psi),

S_c = Schmidt number,

d = diameter of the membrane tube (ft),

Re = Reynolds number,

x = mass fraction of salt in the brine stream,

D_a = diffusivity of NaCl in water (cm^2/sec.).

This equation can be written as

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3-3. Inlet and Exit Brine Concentrations of MS^n, x_i^n and x_e^n.

From the steady state material balance for an infinitesimal element of the membrane tube as shown in Fig. 3 we obtain

$$dq = -dW = -F\rho dS \tag{8}$$

salt material balance for the same element gives

$$xq = (q + dq)(x + dx)$$

$$= sq + qdx + xdq + dxdq$$

if it is assumed that no salt passes through the membrane.

Neglecting the term dxdq yields

$$\frac{dx}{x} = -\frac{dq}{q} \tag{9}$$

Integration of equation (9) from the inlet of the tube to an arbitrary point along the tube gives

$$q = \frac{x_i\ q_i}{x} \tag{10}$$

where the i subscript represents the quantity of the inlet stream.

Substituting equations (8) and (10) into equation (9) yields

$$x_i\ q_i\ \frac{dx}{x^2} = F\rho dS \tag{11}$$

Substituting equation (7) into equation (11) yields

$$x_i q_i \frac{1 + \dfrac{x}{(Re)^{7/8}}}{x^2\ (K\Delta P + bx)}\ dx = \rho dS$$

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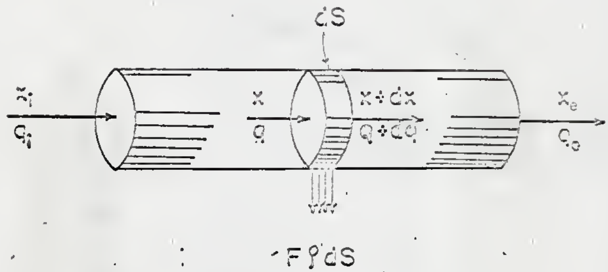

Fig. 3. A tubular reactor representation inside the membrane separator.

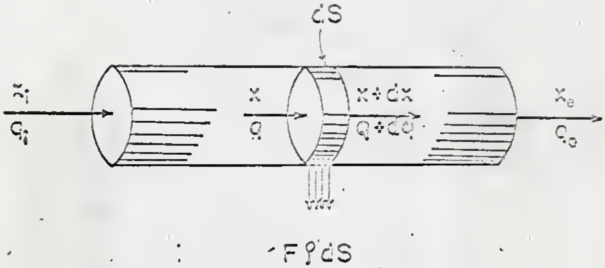

Fig. 3. A tubular reactor representation inside the membrane separator.

Integrating the above equation from x_i to x_e and from 0 to S gives

$$x_i q_i \left\{ \left(\frac{c}{K(Re)^{7/8}\Delta P} - \frac{b}{K^2 \Delta P^2} \right) \ln \frac{x_e (K\Delta P + b x_i)}{x_i \ (K\Delta P + b x_e)} + \frac{1}{K\Delta P} \left(\frac{1}{x_i} - \frac{1}{x_e} \right) \right\} = \rho S \quad (12)$$

In carrying out this integration, the values of ΔP and Re were assumed constant in the range of the whole tube.

After changing the notation from x_i to x_i^n, q_i to q_i^n, ΔP to ΔP^n, Re to Re^n, and x_e to x_e^n, we have the following equation for the n-th stage,

$$x_i^n q_i^n \left\{ \left(\frac{c}{K(Re^n)^{7/8}\Delta P^n} - \frac{b}{K^2(\Delta P^n)^2} \right) \ln \frac{x_e^n (K\Delta P^n + b\, x_i^n)}{x_i^n (K\Delta P^n + b x_e^n)} + \frac{1}{K\Delta P^n} \left(\frac{1}{x_i^n} - \frac{1}{x_e^n} \right) \right\} = \rho S^n \quad (13)$$

3-4. Outlet Brine Concentrations Between Stages, x^n and x^{n-1}.

In Fig. 2 a material balance around the mixing point M^n is

$$q_i^n = (1 + r^n) q^{n-1} \quad (14)$$

salt material balance around point M^n is

$$x^{n-1}\, q^{n-1} + x^n r^n q^{n-1} = x_i^n\ q_i^n \quad (15)$$

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Substituting equations (14), (16) and (3) into equation (13) and noting that $x_e^n = x^n$, we then have

$$(x^{n-1} + r^n x^n) \left\{ \left(\frac{c}{K(Re^n)^{7/8} \Delta P^n} - \frac{b}{K^2(\Delta P^n)^2} \right) \right.$$

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$$= \frac{\rho}{x^o} x^{n-1} \left(\frac{S^n}{q^o} \right) \tag{17}$$

3-5. Reynolds Number Re^n and the Recycle Ratio r^n.

The cross-sectional area through which the brine stream passes at the n-th stage, A^n is given by

$$A^n = \frac{m^n \pi (d)^2}{4} \tag{18}$$

where m^n is the number of tubes in the n-th stage.

The fluid velocity inside the tubes of the n-th stage is

$$u^n = \frac{q_i^n}{A^n \rho} \tag{19}$$

$$= \frac{4q^{n-1}(1 + r^n)}{m^n \pi d^2 \mu}$$

The Reynolds' number is defined by

$$Re^n = \frac{du^n \rho}{\mu}$$

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The membrane area S^n is given by

$$S^n = m^n d\pi L$$

where L is the length of the separator unit.

Substituting the above equation and equation (3) into equation (20) yields

$$Re^n = \frac{4q^o x^o d}{\mu} \left(\frac{L}{d}\right) \frac{(1 + r^n)}{S^n x^{n-1}} \tag{21}$$

As is mentioned before, the Reynolds number Re is assumed to be constant inside a stage in the derivation of equations (12) and (13). From equations (19) and (20) one can see that Re^n is defined as the value of Re at the inlet of the membrane separator in the n-th stage. If higher accuracy is required or percentage conversion of brine to water in any stage becomes very high, some other representation of Re^n such as the average value of Re between the inlet and outlet in the stage must be made.

3-6. Energy Requirement for the High-pressure Pump J_1^n in the n-th Stage, E_1^n

The pumping work E_1^n is primarily used to increase the pressure from P^{n-1} to P^n. Since the velocity difference between the two successive stages is small, the kinetic energy losses and friction losses can be included in the pump efficiency. Thus the power requirement for the high-pressure pump at the n-th stage can be written as

$$E_1^n = \frac{1 + \eta_f}{\eta_m \ \eta_p} \ \frac{p^n - p^{n-1}}{\rho} \ q^{n-1}$$

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Substituting the above equation and equation (3) into equation (20) yields

$$Re^n = \frac{4q^o x^o d}{\mu} \left(\frac{L}{d}\right) \frac{(1 + r^n)}{S^n x^{n-1}} \tag{21}$$

As is mentioned before, the Reynolds number Re is assumed to be constant inside a stage in the derivation of equations (12) and (13). From equations (19) and (20) one can see that Re^n is defined as the value of Re at the inlet of the membrane separator in the n-th stage. If higher accuracy is required or percentage conversion of brine to water in any stage becomes very high, some other representation of Re^n such as the average value of Re between the inlet and outlet in the stage must be made.

3-6. Energy Requirement for the High-pressure Pump J_1^n in the n-th Stage, E_1^n

The pumping work E_1^n is primarily used to increase the pressure from P^{n-1} to P^n. Since the velocity difference between the two successive stages is small, the kinetic energy losses and friction losses can be included in the pump efficiency. Thus the power requirement for the high-pressure pump at the n-th stage can be written as

$$E_1^n = \frac{1 + \eta_f}{\eta_m \, \eta_P} \; \frac{P^n - P^{n-1}}{\rho} \; q^{n-1}$$

where η_m, η_p and η_f are the mechanical and pump efficiency and the friction loss factor.

Substituting equation (3) into the above equation and noting that

$$P^n - P^{n-1} = (P^n - P^o) - (P^{n-1} - P^o) = \Delta P^n - \Delta P^{n-1},$$

we obtain

$$E_1^n = \frac{1 + \eta_f}{\eta_m \eta_p} \frac{\Delta P^n - \Delta P^{n-1}}{\rho} \frac{q^o x^o}{x^{n-1}} \tag{22}$$

Thus, the energy requirement for the high-pressure pump at the n-th stage per unit water production can be given in terms of the brine concentration as

$$\frac{E_1^n}{W_f} = \frac{1 + \eta_f}{\eta_m \eta_p} \frac{\Delta P^n - \Delta P^{n-1}}{\rho} \frac{x^o}{x^{n-1}(1 - \frac{x^o}{x^N})} \tag{23}$$

3-7. Energy Requirement for the Recirculation Pump J_2^n in the n-th Stage, E_2^n

The energy required, E_2^n, includes the energy of circulating $q^{n-1}r^n$ lb_m/hr of the recycle brine and that of the q^{n-1} flow work. The friction loss comes largely from the fluid flowing in the membrane separator unit. This lost work based on unit time is

$$E_2^n = 4f \frac{(u^n)^2}{2g_c} \left(\frac{L}{d}\right) q_i^n \frac{1 + \eta_f}{\eta_m \eta_p} \tag{24}$$

where f is the friction factor.

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Substituting equation (3) into the above equation and noting that

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where f is the friction factor.

From rearranging equation (19), the flow rate of the brine stream entering the membrane separator in the n-th stage can be written as follows

$$q_1^n = m^n \frac{\pi d^2}{4} u^n \rho \tag{25}$$

For turbulent flow the friction factor can be approximated by

$$f = \frac{0.046}{(Re^n)^{0.2}}$$

Substituting the above equation and equation (25) into equation (24) yields

$$E_2^n = 0.023 \frac{1 + \eta_f}{\eta_m \eta_p} \frac{\rho}{g_c} \left(\frac{\mu}{d\rho}\right)^3 (Re^n)^{2.8} S^n \tag{26}$$

where $S^n = m^n L \pi d$, the membrane area in the n-th stage.

The energy requirement per unit water production is

$$\frac{E_2^n}{W_f} = 0.023 \frac{1 + \eta_f}{\eta_m \eta_p} \frac{\rho}{g_c} \left(\frac{\mu}{d\rho}\right)^3 (Re^n)^{2.8} \frac{S^n}{q^o\left(1 - \frac{x^o}{x^N}\right)} \tag{27}$$

3-8. Energy Recovery at the Reject Brine Turbine, E_3

The energy recovery from depressurizing the high-pressure brine stream from P^N to P^o (discharge pressure) is given by

$$E_3 = \eta_m \eta_p (1 - \eta_f) \frac{P^N - P^o}{\rho} q^N$$

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Noting that $P^N - P^o = \Delta P^N$ and substituting equation (3) into the above equation, we obtain

$$E_3 = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o q^o}{\dot{x}^N} \tag{28}$$

The energy recovery per unit water production can be written as

$$\frac{E_3}{W_f} = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o}{x^N - x^o} \tag{29}$$

3-9. Simplified Models

Three simplified models A, B, and C are presented here. Model A is essentially similar to the general model except that it has the same membrane area in each stage (i.e. $S^n = S$). Model B is a sequential system without recirculation pump in each stage (i.e. $r^n = 0$) and is depicted in Fig. 4. Model C is a sequential system with only one pump in the first stage and is shown in Fig. 5.

The model A has been suggested by the fact that it is often economical to use an identical unit at each stage for a multistage system.

For a sequential multi-stage system, if the cost function is in the linear form, the system with recycle operation is often an optimal configuration (23, 28). However, if the cost function is in the non-linear form, this may not be true. This has given rise to model B. It also appears that it may not be necessary to use a high-pressure pump in each stage but just to let the pressure of the brine stream decrease successively in each stage. Therefore, the model C, which is simpler than the model B is proposed.

Noting that $P^N - P^o = \Delta P^N$ and substituting equation (3) into the above equation, we obtain

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The energy recovery per unit water production can be written as

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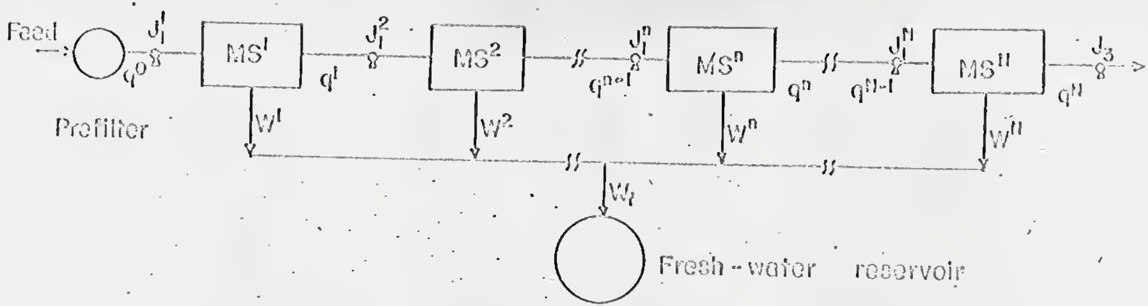
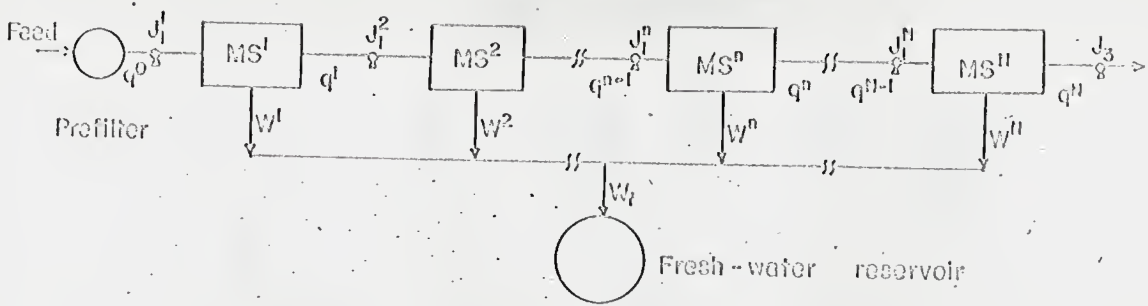


Fig. 4. Schmatic diagram of a sequential N-stage reverse osmosis water purification process of model D.



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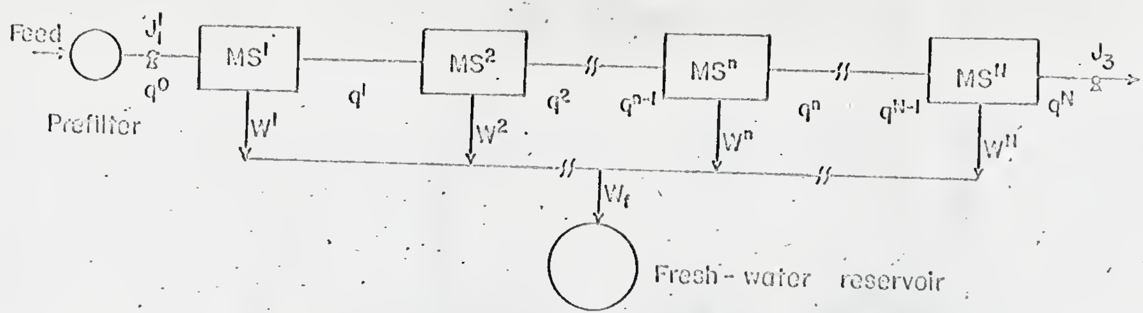

Fig. 5. Schematic diagram of a sequential N-stage reverse osmosis water purification process of model C.

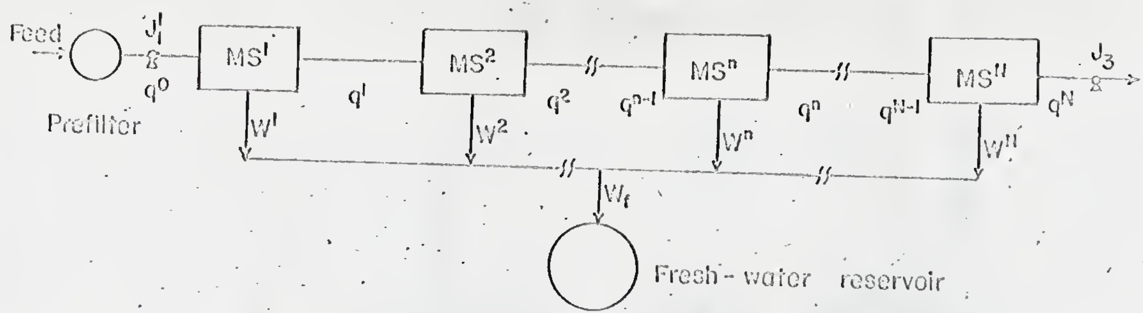

Fig. 5. Schematic diagram of a sequential N-stage reverse osmosis water purification process of model C.

Reasons for proposing different models may not be entirely valid. They can only be verified through an optimization study of each model using both linear and non-linear representations of the cost function.

The energy requirements for each model are derived below. The second subscripts a, b, and c in the various energy terms, E^n_{ia}, E^n_{ib}, and E^n_{ic} are used to represent the various energy terms of the models A, B, and C respectively. There is no such additional subscript attached to the general model.

Model A.

The basic assumption of this model is to use the same number of tubes in each stage. Since

$$m^n = m \ , \quad n = 1, 2, \ldots\ldots N \ ,$$

$$S^n = m^n d\pi L = md\pi L = S \ , \quad n = 1, 2, \ldots\ldots N \tag{30}$$

$$A^n = \frac{m^n\pi(d)^2}{4} = \frac{m\pi(d)^2}{4} = A, \ n = 1, 2, \ldots\ldots N \tag{31}$$

After changing S^n to S, equations (17), (21), (26), and (27) become

$$(x^{n-1} + r^n x^n)\left\{\left(\frac{c}{K(Re^n)^{7/8}\Delta P^n} - \frac{b}{K^2(\Delta P^n)^2}\right) \cdot \ln \frac{x^n\left[K\Delta P^n(1+r^n) + b(x^{n-1} + r^n x^n)\right]}{(x^{n-1} + r^n x^n)(K\Delta P^n + bx^n)} + \frac{1}{K\Delta P^n}\left(\frac{1 + r^n}{x^{n-1} + r^n x^n} - \frac{1}{x^n}\right)\right\}$$

$$= \frac{\rho}{x^o}\, x^{n-1}\left(\frac{S}{q^o}\right) \tag{32}$$

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$$= \frac{\rho}{x^o}\ x^{n-1}\left(\frac{S}{q^o}\right) \tag{32}$$

and

$$Re^n = \frac{4q^o x^o d}{\mu} \left(\frac{L}{d}\right) \frac{(1 + r^n)}{S\ x^{n-1}} \tag{33}$$

and

$$E^n_{2a} = 0.023\ \frac{1 + \eta_f}{\eta_m \eta_p}\ \frac{\rho}{g_c}\ \left(\frac{\mu}{d\rho}\right)^3\ (Re^n)^{2.8}\ S \tag{34}$$

and

$$\frac{E^n_{2a}}{W_f} = 0.023\ \frac{1 + \eta_f}{\eta_m \eta_p}\ \frac{\rho}{g_c}\ \left(\frac{\mu}{d\rho}\right)^3\ (Re^n)^{2.8}\ \frac{S}{q^o\left(1 - \frac{x^o}{x^N}\right)} \tag{35}$$

respectively.

Equations (22), (23), (28), and (29) are still valid for this model. Therefore,

$$E^n_{1a} = E^n_1 = \frac{1 + \eta_f}{\eta_m \eta_p}\ \frac{\Delta P^n - \Delta P^{n-1}}{\rho}\ \frac{q^o x^o}{x^{n-1}} \tag{36}$$

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$$E_{3a} = E_3 = \eta_m \eta_p\ (1 - \eta_f)\ \frac{\Delta P^N}{\rho}\ \frac{x^o q^o}{x^N} \tag{38}$$

$$\frac{E_{3a}}{W_f} = \frac{E_3}{W_f} = \eta_m \eta_p\ (1 - \eta_f)\ \frac{\Delta P^N}{\rho}\ \frac{x^o}{x^N - x^o} \tag{39}$$

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Model B

As is shown in Fig. 4, only a high-pressure pump is used in each stage instead of a high pressure pump and a recirculation pump. The velocity of the brine stream in each stage may be adjusted by using a different membrane area in each stage.

After dropping the recycle ratio r^n in equations (17) and (21) we obtain

$$\left\{ \left(\frac{c}{K(Re^n)^{7/8} \Delta P^n} - \frac{b}{K^2(\Delta P^n)^2} \right) \ln \frac{x^n(K\Delta P^n + bx^{n-1})}{x^{n-1}(K\Delta P^n + bx^n)} + \frac{1}{K\Delta P^n} \left(\frac{1}{x^{n-1}} - \frac{1}{x^n} \right) \right\} = \frac{\rho}{x^o} \left(\frac{S^n}{q^o} \right) \tag{40}$$

and

$$Re^n = \frac{4q^o x^o d}{\mu} \left(\frac{L}{d} \right) \frac{1}{S^n x^{n-1}} \tag{41}$$

Equations (28) and (29) are still correct for this model. Therefore, we have

$$E_{3b} = E_3 = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o q^o}{x^N} \tag{42}$$

$$\frac{E_{3b}}{W_f} = \frac{E_3}{W_f} = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o}{x^N - x^o} \tag{43}$$

For this model, the energy requirement for the high pressure pump in the n-th stage E_{1b}^n includes not only the energy used to increase the pressure from P^{n-1} to P^n but also that of the pumping head to overcome friction in the n-th stage. Therefore, the following expressions are adequate

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$$E_{3b} = E_3 = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o q^o}{x^N} \tag{42}$$

$$\frac{E_{3b}}{W_f} = \frac{E_3}{W_f} = \eta_m \eta_p (1 - \eta_f) \frac{\Delta P^N}{\rho} \frac{x^o}{x^N - x^o} \tag{43}$$

For this model, the energy requirement for the high pressure pump in the n-th stage E_{1b}^n includes not only the energy used to increase the pressure from P^{n-1} to P^n but also that of the pumping head to overcome friction in the n-th stage. Therefore, the following expressions are adequate

$$E_{1b}^{n} = E_{1}^{n} + E_{2}^{n}$$

$$= \frac{1+\eta_f}{\eta_m\eta_p}\ \frac{\Delta P^{n} - \Delta P^{n-1}}{\rho}\ \frac{q^{o}x^{o}}{x^{n-1}}$$

$$+ 0.023\ \frac{1+\eta_f}{\eta_m\eta_p}\ \frac{\rho}{g_c}\ \left(\frac{\mu}{d\rho}\right)^{3}\ (Re^{n})^{2.8}\ S^{n} \qquad (44)$$

$$\frac{E_{1b}^{n}}{W_f} = \frac{E_{1}^{n}}{W_f} + \frac{E_{2}^{n}}{W_f}$$

$$= \frac{1+\eta_f}{\eta_m\eta_p}\ \frac{\Delta P^{n} - \Delta P^{n-1}}{\rho}\ \frac{x^{o}}{x^{n-1}\left(1 - \frac{x^{o}}{x^{N}}\right)}$$

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$$E_{2b}^{n} = 0 \qquad (46)$$

$$\frac{E_{2b}^{n}}{W_f} = 0 \qquad (47)$$

Model C

Model C is shown in Fig. 5. In this model only one high pressure pump is used in the first stage, i.e., no pumps are used in the remaining stages. Since pressure changes along the tube in each stage, an exact solution involves a complicated intergration. An approximate solution can be obtained if we assume constant pressure inside each stage but changes from stage to stage abruptly. Therefore equations (40) and (41) of model B are adequate here, but the pressures between the n-th and the (n-1)-th stage can be related by the following relation: (26)

$$E_{1b}^{n} = E_1^n + E_2^n$$

$$= \frac{1+\eta_f}{\eta_m \eta_p} \; \frac{\Delta P^n - \Delta P^{n-1}}{\rho} \; \frac{q^o x^o}{x^{n-1}}$$

$$+ \; 0.023 \, \frac{1+\eta_f}{\eta_m \eta_p} \; \frac{\rho}{g_c} \left(\frac{\mu}{d\rho}\right)^3 \; (Re^n)^{2.8} \; S^n \qquad (44)$$

$$\frac{E_{1b}^{n}}{W_f} = \frac{E_1^n}{W_f} + \frac{E_2^n}{W_f}$$

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$$+ \; 0.023 \, \frac{1+\eta_f}{\eta_m \eta_p} \; \frac{\rho}{g_c} \left(\frac{\mu}{d\rho}\right)^3 \; (Re^n)^{2.8} \; \frac{S^n}{q^o\left(1 - \frac{x^o}{x^N}\right)} \qquad (45)$$

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$$\Delta P^n = \Delta P^{n-1} - H_{fs}\rho \tag{48}$$

where

$$H_{fs} = 4f\frac{L}{d}\frac{(u^n)^2}{2g_c} ,$$

Substituting the friction factor as given in Section 3-7 into the above equation yields

$$\Delta P^n = \Delta P^{n-1} - 0.092\left(\frac{L}{d}\right)\frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^2 (\mathrm{Re}^n)^2 \tag{49}$$

The energy equations necessary for this model are

$$E_{1c}^1 = E_1^1 = \frac{1+\eta_f}{\eta_m\eta_p}\ \frac{\Delta P^1}{\rho}\ q^o \tag{50}$$

$$E_{1c}^n = 0, \qquad n = 2, 3, \dots\dots\dots, N \tag{51}$$

$$E_{2c}^n = 0, \qquad n = 1, 2, 3, \dots\dots\dots, N \tag{52}$$

$$E_{3c} = E_3 = \eta_p\eta_m(1-\eta_f)\ \frac{\Delta P^N}{\rho}\ \frac{x^o q^o}{x^N} \tag{53}$$

$$\frac{E_{1c}^1}{W_f} = \frac{E_1^1}{W_f} = \frac{1+\eta_f}{\eta_m\eta_p}\ \frac{\Delta P^1}{\rho}\ \frac{1}{\left(1-\frac{x^o}{x^N}\right)} \tag{54}$$

$$\frac{E_{1c}^n}{W_f} = 0, \qquad n = 2, 3, 4, \quad \dots\dots\dots\dots, N \tag{55}$$

$$\frac{E_{2c}^n}{W_f} = 0, \qquad n = 1, 2, 3, \quad \dots\dots\dots\dots, N \tag{56}$$

$$\frac{E_{3c}}{W_f} = \frac{E_3}{W_f} = \eta_p\,\eta_m\,(1-\eta_f)\ \frac{\Delta P^N}{\rho}\ \frac{x^o}{x^N - x^o} \tag{57}$$

$$\Delta P^n = \Delta P^{n-1} - H_{fs}\rho \tag{48}$$

where

$$H_{fs} = 4f\frac{L}{d}\frac{(u^n)^2}{2g_c},$$

Substituting the friction factor as given in Section 3-7 into the above equation yields

$$\Delta P^n = \Delta P^{n-1} - 0.092\left(\frac{L}{d}\right)\frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^2 (Re^n)^2 \tag{49}$$

The energy equations necessary for this model are

$$E_{1c}^1 = E_1^1 = \frac{1+\eta_f}{\eta_m\eta_p}\;\frac{\Delta P^1}{\rho}\;q^o \tag{50}$$

$$E_{1c}^n = 0, \qquad n = 2, 3, \ldots\ldots\ldots, N \tag{51}$$

$$E_{2c}^n = 0, \qquad n = 1, 2, 3, \ldots\ldots\ldots, N \tag{52}$$

$$E_{3c} = E_3 = \eta_p\eta_m(1-\eta_f)\;\frac{\Delta P^N}{\rho}\;\frac{x^o q^o}{x^N} \tag{53}$$

$$\frac{E_{1c}^1}{W_f} = \frac{E_1^1}{W_f} = \frac{1+\eta_f}{\eta_m\eta_p}\;\frac{\Delta P^1}{\rho}\;\frac{1}{\left(1-\frac{x^o}{x^N}\right)} \tag{54}$$

$$\frac{E_{1c}^n}{W_f} = 0, \qquad n = 2, 3, 4, \ldots\ldots\ldots\ldots, N \tag{55}$$

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CHAPTER IV

ECONOMIC ANALYSIS

Cost of the plant may be divided into the two major parts: the capital cost and the operating cost. The capital cost consists of three items:

(a) Pump cost,

(b) Turbine cost,

(c) Membrane area cost.

The operating cost includes four items:

(a) Power cost for the high pressure pump,

(b) Power cost for the circulation pump,

(c) Energy recovery from the reject turbine,

(d) Feed brine cost.

Other costs such as labor cost, insurance cost, etc., are not considered here as they have little effect on the water cost when the operating conditions are changed.

The symbols which represent the cost items mentioned above are listed below. The first subscripts are referred to the various cost items and the second the various models.

C_1, C_{1a}, C_{1b}, C_{1c} = the capital costs of the pumps for the general model, and models A, B, and C, respectively.

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C_3, C_{3a}, C_{3b}, C_{3c} = the capital cost of the membrane separator unit for the general model, and models A, B, and C, respectively.

C_4, C_{4a}, C_{4b}, C_{4c} = the power cost of the high pressure pump for the general model, and models A, B, and C, respectively.

C_5, C_{5a}, C_{5b}, C_{5c} = the power costs of the circulation pump for the general model, and models A, B, and C, respectively.

C_6, C_{6a}, C_{6b}, C_{6c} = the energy costs recovered from the reject turbine for the general model, and model A, B, and C, respectively.

C_7, C_{7a}, C_{7b}, C_{7c} = the feed brine costs for the general model, and models A, B, and C, respectively.

C_t, C_{ta}, C_{tb}, C_{tc} = the total water costs per unit mass of production for the general model, and models A, B, and C, respectively.

4-1. The Capital Cost

The annual capitalization charge for the equipment items is taken to be 0.074 of the initial cost per year, as recommended in the Office of Saline Water Report (12). An assumption of a load factor of 330-on-stream day per year gives a capitalization charge, ψ, of 9.4×10^{-6} of the initial cost per hour on stream.

The power rule relating the capital cost and the equipment capacity as suggested by Chilton (27) is

$$C = k\,T^{a} \qquad (58)$$

$C_3, C_{3a}, C_{3b}, C_{3c}$ = the capital cost of the membrane separator unit for the general model, and models A, B, and C, respectively.

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C = the capital cost of the equipment

T = the capacity of the equipment

k = a proportionality constant

a = a positive constant less than 1.0.

This rule is used where the capital costs of the equipment are concerned.

(a) Pump and Turbine Cost, C_1 and C_2

The proportionality constants in the power rule for the high pressure pump, recirculation pump, and turbine are represented respectively by k_1, k_2, and k_t. According to the power rule mentioned above, the pump and the turbine costs are given respectively by

$$C_1 = \psi \sum_{n=1}^{N} \left\{ \frac{k_1 (E_1^n)^{a_1} + k_2 (E_2^n)^{a_2}}{W_f} \right\} \tag{59}$$

and

$$C_2 = \psi k_t \frac{(E_3)^{a_3}}{W_f} \tag{60}$$

where a_1, a_2, and a_3 are the power rule coefficients for the high pressure pump, the circulation pump, and turbine, respectively.

(B) Membrane Separator Cost, C_3

The mass of the membrane separator for the n-th stage is given by (20)

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(B) Membrane Separator Cost, C_3

The mass of the membrane separator for the n-th stage is given by (20)

$$W_s^n = \frac{\rho_m d}{\sigma_m} S^n \Delta P^n \left(1.62 + \frac{0.54}{L/D} + \frac{0.189}{L/D}\sqrt{\frac{\sigma_m}{\Delta P^n}}\right) \tag{61}$$

and

$$W_s^n = \frac{\rho_m d}{\sigma_m} \frac{S^n \Delta P^n}{q^o(1 - \frac{x^o}{x^N})} \left(1.62 + \frac{0.54}{L/D} + \frac{0.189}{L/D}\sqrt{\frac{\sigma_m}{\Delta P^n}}\right) \tag{62}$$

where

W_s^m = the mass of the membrane separator for the n-th stage (lb_m),

ρ_m = the density of the material of the construction (lb_m/ft^3),

σ_m = the allowable stress of the material of construction (psi),

L/D = the overall length-to-diameter ratio of the membrane separator

The proportionality constant in the power rule for membrane separator is represented by k_s. The membrane separator cost, C_3, can then be written as

$$C_3 = k_s \sum_{n=1}^{N} \frac{(W_s^n)^{a_4}}{W_f} \qquad (\$/lbm) \tag{63}$$

where a_4 is the power rule constant for the membranc separator.

Since we have assumed certain constant values for L, d, and $(\frac{L}{D})$, this gives rise to an inequality constraint, $(\frac{L}{d})D^2\pi > S^n$ that must be satisfied in the selection of S^n.

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The unit electrical power cost is represcnted by C_e, \$/psi-ft^3. The power cost for the high pressure pump C_4, for the circulation pump C_5, and the energy cost recovered from the reject turbine C_6 arc given by

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and

$$C_6 = C_e \frac{E_3}{W_f} \quad (\$/lbm) \tag{66}$$

respectively.

The cost of the brine feed per unit water production can be given by

$$C_7 = C_F \frac{q^o}{W_f} \quad (\$/lbm) \tag{67}$$

where C_F is the unit cost of the brine feed.

Substituting equation (5) into the above equation yields

$$C_7 = C_F \frac{x^N}{x^N - x^o} \tag{68}$$

4-3. The Water Cost, C_t

The total water cost per unit water production is the sum of the various cost items, i.e.,

$$C_t = C_1 + C_2 + C_3 + C_4 + C_5 - C_6 + C_7$$

$$C_4 = C_e \sum_{n=1}^{N} \frac{E_1^n}{W_f} \quad (\$/lbm) \tag{64}$$

and

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4-4. Water Costs for Models A, B, and C

The derivation of the cost equations for models A, B, and C is similar to that for the general model. The cost equations for the various models are listed below:

(A) Model A

$$C_{1a} = \psi \sum_{n=1}^{N} \left\{ \frac{k_1 (E_{1a}^n)^{a_1} + k_2 (E_{2a}^n)^{a_2}}{W_f} \right\} \tag{70}$$

$$C_{2a} = \psi k_t \frac{(E_{3a})^{a_3}}{W_f} \tag{71}$$

$$C_{3a} = \psi k_s \sum_{n=1}^{N} \frac{(W_{sa}^n)^{a_4}}{W_f} \tag{72}$$

where

$$W_{sa}^n = \frac{\rho_m d}{\sigma_m} S \Delta P^n \left(1.62 + \frac{0.54}{L/D} + \frac{0.189}{L/D} \sqrt{\frac{\sigma_m}{\Delta P^n}}\right) \tag{73}$$

$$C_{4a} = C_e \sum_{n=1}^{N} \frac{E_{1a}^n}{W_f} \tag{74}$$

$$C_{5a} = C_e \sum_{n=1}^{N} \frac{E_{2a}^n}{W_f} \tag{75}$$

$$C_{6a} = C_e \frac{E_{3a}}{W_f} \tag{76}$$

$$C_{7a} = C_F \frac{x^N}{x^N - x^o} \tag{77}$$

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(B) Model B

$$C_{1b} = \psi k_1 \sum_{n=1}^{N} \frac{(E_{1b}^{n})^{a_1}}{W_f} \tag{79}$$

$$C_{2b} = \psi k_t \frac{(E_{3b})^{a_3}}{W_f} \tag{80}$$

$$C_{3b} = \psi k_s \sum_{n=1}^{N} \frac{(W_s^{n})^{a_4}}{W_f} \tag{81}$$

$$C_{4b} = C_e \sum_{n=1}^{N} \frac{E_{1b}^{n}}{W_f} \tag{82}$$

$$C_{5b} = 0 \tag{83}$$

$$C_{6b} = C_e \frac{E_{3b}}{W_f} \tag{84}$$

$$C_{7b} = C_F \frac{x^N}{x^N - x^o} \tag{85}$$

$$C_{tb} = C_{1b} + C_{2b} + C_{3b} + C_{4b} - C_{6b} + C_{7b} \tag{86}$$

(C) Model C

$$C_{1c} = \psi \; k_1 \frac{(E_{1c}^{1})^{a_1}}{W_f} \tag{87}$$

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CHAPTER 5

OPTIMIZATION

Equations (17) and (21) show the relations among the various quantities of the n-th stage such as the brine concentration x^n, the recycle ratio r^n, the pressure difference across the membrane ΔP^n, the Reynolds number Re^n, and the membrane area S^n. The water production cost is a function of these 5N variables and is given by equation (69). However, the 2N relationships given by equations (17) and (21) reduce the number of independent variables from 5N to 3N. Since the brine concentration leaving the last stage, x^N, must be prefixed in order to calculate the energy requirements in each stage, the total number of independent variables becomes 3N-1.

A discrete version of the maximum principle is powerful for searching the optimum condition of a multi-stage multi-decision process. For the process in hand, which is an N-stage 3-decision process, the performance equations are summarized in section 5-1. The derivatives of the state variables are listed in section 5-2. The adjoint variables and derivatives of the Hamiltonian functions are determined respectively in sections 5-3 and 5-4. The computing procedures are described in section 5-5.

Similarly, the same procedures can be applied to the simplified models.

5-1. Performance Equations

Equation (17) can be rewritten with the aid of equation (5) as

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5-1. Performance Equations

Equation (17) can be rewritten with the aid of equation (5) as

$$(x^{n-1}+r^n x^n)\left\{\left(\frac{c}{(Re)^{7/8}\,\Delta P^n} - \frac{b}{K^2(\Delta P^n)^2}\right) \ln \frac{x^n\left\{K\Delta P^n(1+r^n) + b(x^{n-1}+r^n x^n)\right\}}{(x^{n-1} + r^n x^n)(K\Delta P^n + bx^n)}\right.$$

$$\left. + \frac{1}{K\Delta P}\left(\frac{1 + r^n}{x^{n-1} + r^n x^n} - \frac{1}{x^n}\right)\right\} = \frac{\rho}{W_f x^o}\left(1 - \frac{x^o}{x^N}\right) x^{n-1}\, S^n \qquad (95)$$

$$n = 1, \ldots, N$$

Rearrangement of equation (21) gives

$$r^n = \frac{\mu\left(1 - \frac{x^o}{x^N}\right)x^{n-1}\, Re^n\, S^n}{4\, x^0\, d(L/d)\, W_f} - 1 \qquad n = 1, \ldots, N \qquad (96)$$

Substituting the various equations into equation (69) yields

$$C_t = \frac{\psi}{W_f}\sum_{n=1}^{N}\left\{k_1\left(\frac{1 + \eta_f}{\eta_m\,\eta_p}\cdot\frac{\Delta P^n - \Delta P^{n-1}}{\rho}\;\frac{x^0\, W_f}{\left(1-\frac{x^0}{x^o}\right)x^{n-1}}\right)^{a_1}\right.$$

$$\left. + k_2\left[0.023\,\frac{1 + \eta_f}{\eta_m\,\eta_p}\;\frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^3 (Re^n)^{2.8}\, S^n\right]^{a_2}\right\}$$

$$+ \frac{\psi k_t}{W_f}\left[\eta_m\eta_p\,(1 - \eta_f)\;\frac{\Delta P^N}{\rho}\;\frac{x^0\, W_f}{\left(1 - \frac{x^0}{x^N}\right)x^N}\right]^{a_3}$$

$$+ \frac{k_s}{W_f}\sum_{n=1}^{N}\left[\frac{\rho_m d}{\sigma_m}\, S^n\,\Delta P^n\left(1.62 + \frac{0.54}{L/D} + \frac{0.189}{L/D}\sqrt{\frac{\sigma_m}{\Delta P^n}}\right)\right]^{a_4}$$

$$+ C_e\sum_{n=1}^{N}\left[\frac{1 + \eta_f}{\eta_m\,\eta_p}\;\frac{\Delta P^n - \Delta P^{n-1}}{\rho}\;\frac{x^o}{x^{n-1}\left(1 - \frac{x^o}{x^o}\right)} + 0.023\;\frac{1 + \eta_f}{\eta_m\,\eta_p}\,\frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^3\right.$$

$$\left.(Re^n)^{2.8}\,\frac{S^n}{W_f}\right] - C_e\,\eta_m\,\eta_P\,(1 - \eta_f)\;\frac{\Delta P^N}{\rho}\,\frac{x^o}{x^N - x^o} + C_F\cdot$$

$$\frac{x^o}{x^N - x^o} \qquad (97)$$

$$(x^{n-1}+r^n x^n)\left\{\left(\frac{c}{(Re)^{7/8}\,\Delta P^n} - \frac{b}{K^2(\Delta P^n)^2}\right) \ln \frac{x^n\left\{K\Delta P^n(1+r^n) + b(x^{n-1}+r^n x^n)\right\}}{(x^{n-1} + r^n x^n)(K\Delta P^n + bx^n)}\right.$$

$$\left. + \frac{1}{K\Delta P}\left(\frac{1 + r^n}{x^{n-1} + r^n x^n} - \frac{1}{x^n}\right)\right\} = \frac{\rho}{W_f x^o}\left(1 - \frac{x^o}{x^N}\right) x^{n-1}\, S^n \quad (95)$$

$$n = 1, \ldots, N$$

Rearrangement of equation (21) gives

$$r^n = \frac{\mu\left(1 - \frac{x^o}{x^N}\right)x^{n-1}\, Re^n\, S^n}{4\, x^O\, d(L/d)\, W_f} - 1 \qquad n = 1, \ldots, N \quad (96)$$

Substituting the various equations into equation (69) yields

$$C_t = \frac{\psi}{W_f} \sum_{n=1}^{N} \left\{ k_1\left(\frac{1 + \eta_f}{\eta_m \eta_p} \cdot \frac{\Delta P^n - \Delta P^{n-1}}{\rho} \; \frac{x^O\, W_f}{\left(1 - \frac{x^O}{x^o}\right)x^{n-1}}\right)^{a_1}\right.$$

$$\left. + k_2\left[0.023\, \frac{1 + \eta_f}{\eta_m \eta_p}\, \frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^3 (Re^n)^{2.8}\, S^n\right]^{a_2}\right\}$$

$$+ \frac{\psi k_t}{W_f}\left[\eta_m \eta_p\, (1 - \eta_f)\, \frac{\Delta P^N}{\rho}\, \frac{x^O\, W_f}{\left(1 - \frac{x^O}{x^N}\right)x^N}\right]^{a_3}$$

$$+ \frac{k_s}{W_f} \sum_{n=1}^{N} \left[\frac{\rho_m d}{\sigma_m}\, S^n\, \Delta P^n \left(1.62 + \frac{0.54}{L/D} + \frac{0.189}{L/D}\sqrt{\frac{\sigma_m}{\Delta P^n}}\right)\right]^{a_4}$$

$$+ C_e \sum_{n=1}^{N} \left[\frac{1 + \eta_f}{\eta_m \eta_p}\, \frac{\Delta P^n - \Delta P^{n-1}}{\rho}\, \frac{x^o}{x^{n-1}\left(1 - \frac{x^o}{x^o}\right)} + 0.023\, \frac{1 + \eta_f}{\eta_m \eta_p}\, \frac{\rho}{g_c}\left(\frac{\mu}{d\rho}\right)^3\right.$$

$$\left.(Re^n)^{2.8}\, \frac{S^n}{W_f}\right] - C_e\, \eta_m \eta_P\, (1 - \eta_f)\, \frac{\Delta P^N}{\rho}\, \frac{x^o}{x^N - x^o} + C_F \cdot$$

$$\frac{x^o}{x^N - x^o} \qquad (97)$$

The 3N decision variables and 2N state variables are defined as follows:

$$\theta_1^n = Re^n \qquad n = 1, \ldots, N \tag{98}$$

$$\theta_2^n = \Delta P^n \qquad n = 1, \ldots, N \tag{99}$$

$$\theta_3^n = S^n \qquad n = 1, \ldots, N \tag{100}$$

$$x_1^n = x^n \qquad n = 1, \ldots, N \tag{101}$$

$$x_2^N = C_t \tag{102}$$

After such transformations equations (95) and (96) become

$$(x_1^{n-1} + r^n x_1^n)\left\{\left(\frac{c}{K(\theta_1^n)^{7/8}\,\theta_2^n} - \frac{b}{K^2(\theta_2^n)^2}\right)\ln\frac{x_1^n\left[K\theta_2^n(1+r^n) + b(x_2^{n-1}+r^n x_2^n)\right]}{(x_1^{n-1} + r^n x_1^n)(K\theta_2^n + bx_1^n)}\right.$$

$$\left. + \frac{1}{K\theta_2^n}\left(\frac{1 + r^n}{x_1^{n-1} + r^n x_1^n} - \frac{1}{x_1^n}\right)\right\} = B_1\left(1 - \frac{x_1^o}{x_1^N}\right) x_1^{n-1}\ \theta_3^n \tag{103}$$

where

$$B_1 = \frac{\rho}{x_1^o W_f} \qquad n = 1, \ldots, N$$

and

$$r^n = B_2\left(1 - \frac{x_1^o}{x_1^N}\right)\theta_1^n\ \theta_3^n\ x_1^{n-1} - 1 \qquad n = 1, \ldots, N \tag{104}$$

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where

$$B_2 = \frac{\mu}{4\, x_1^o\, d(L/D)\overline{W}_f}$$

x_2^n is defined as follows:

$$x_2^n = x_2^{n-1} + B_3\left[B_4\ \frac{\theta_2^n - \theta_2^{n-1}}{x_1^{n-1}\left(1 - \frac{x_1^o}{x_1^N}\right)}\right]^{a_1} + B_{13}\left[B_5({}_1^n)^{2.8}\ \theta_3^n\right]^{a_2}$$

$$+ B_6\left[\theta_3^n\ \theta_2^n\left(B_7 + B_8(\theta_2^n)^{-\frac{1}{2}}\right)\right]^{a_4}$$

$$+ B_9\ \frac{\theta_2^n - \theta_2^{n-1}}{x_1^{n-1}\left(1 - \frac{x_1^o}{x_1^N}\right)} + B_{10}(\theta_1^n)^{2.8}\ \theta_3^n \qquad (105)$$

$n = 1, \ldots, N\text{-}1$

$$x_2^N = x_2^{N-1} + B_3\left[B_4\ \frac{\theta_2^N - \theta_2^{N-1}}{x_1^N\left(1 - \frac{x_1^o}{x_1^N}\right)}\right]^{a_1} + B_{13}\left[B_5(\theta_1^N)^{2.8}\ \theta_3^N\right]^{a_2}$$

$$+ B_6\left[\theta_3^N\ \theta_2^N\ \left(B_7 + B_8(\theta_2^N)^{-\frac{1}{2}}\right)\right]^{a_4} + B_9\ \frac{\theta_2^N - \theta_2^{N-1}}{x_1^{N-1}\left(1 - \frac{x_1^o}{x_1^N}\right)}$$

$$+ B_{10}(\theta_1^N)^{2.8}\ \theta_3^N$$

$$+ B_{14}\left[B_{11}\ \frac{\theta_2^N}{x_1^N - x_1^o}\right]^{a_3} - B_{12}\ \frac{\theta_2^N}{x_1^N - x_1^o} + C_F\ \frac{x_1^N}{x_1^N - x_1^o} \qquad (106)$$

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$$+ B_6\left[\theta_3^N\ \theta_2^N\ \left(B_7 + B_8(\theta_2^N)^{-\frac{1}{2}}\right)\right]^{a_4} + B_9\ \frac{\theta_2^N - \theta_2^{N-1}}{x_1^{N-1}\left(1 - \frac{x_1^o}{x_1^N}\right)}$$

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where

$$x_2^o = 0$$

$$B_3 = \frac{\psi k_1}{W_f}$$

$$B_4 = \frac{1 + \eta_f}{\eta_m \eta_p} \cdot \frac{x_1^o W_f}{\rho}$$

$$B_5 = 0.023 \frac{1 + \eta_f}{\eta_m \eta_p} \frac{\rho}{g_c} \left(\frac{\mu}{d\rho}\right)^3$$

$$B_6 = \frac{\psi k_s}{W_f}$$

$$B_7 = \frac{\rho_m d}{\sigma_m} \left(1.62 + \frac{0.54}{L/D}\right)$$

$$B_8 = 0.189 \frac{\rho_m d}{L/D \sqrt{\sigma_m}}$$

$$B_9 = C_e \frac{1 + \eta_f}{\eta_m \eta_p} \frac{x_1^o}{\rho}$$

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Equations (105) and (106) contain the term θ_2^{n-1}, that is, it has decision in memory. To avoid this, x_3^n is introduced as (11)

$$x_3^n = \theta_2^n \qquad n = 1, \ldots, N \tag{107}$$

Then equations (105) and (106) can be rewritten as

$$x_2^n = x_2^{n-1} + B_3 \left[B_4 \frac{\theta_2^n - \theta_3^{n-1}}{x_1^{n-1}\left(1 - \frac{x_1^o}{x_1^N}\right)} \right]^{a_1} + B_{13} \left[B_5 (\theta_1^n)^{2.8} \; \theta_3^n \right]^{a_2}$$

$$+ B_6 \left[\theta_3^n \; \theta_2^n \left(B_7 + B_8 (\theta_2^n)^{-\frac{1}{2}}\right) \right]^{a_4} + B_9 \frac{\theta_2^n - x_3^{n-1}}{x_1^{n-1}\left(1 - \frac{x_1^o}{x_1^N}\right)}$$

$$+ B_{10} (\theta_1^n)^{2.8} \; \theta_3^n \qquad n = 1, \ldots, N \tag{108}$$

$$x_2^N = x_2^{N-1} + B_3 \left[B_4 \frac{\theta_2^N - x_3^{N-1}}{x_1^{N-1}\left(1 - \frac{x_1^o}{x_1^N}\right)} \right]^{a_1} + B_{13} \left[B_5 (\theta_1^N)^{2.8} \; \theta_3^N \right]^{a_2}$$

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Now, the optimization problem may be formulated as this:

Find a set of decision variables θ_1^n, θ_2^n, and θ_3^n (n = 1, 2, ... N) to minimize the water cost x_2^N with x_1^N prefixed.

5-2. Derivatives of State Variables

For convenience the symbols g_n^n and h_n^n are used to represent the various combinations of the state variables x_i^n, decision variables θ_i^n, and constants B_n as defined before. The two symbols are listed respectively in Tables 1 and 2.

(1) x_1^n

$$\frac{\partial x_1^n}{\partial \theta_1^n} = \frac{g_{43}^n}{g_{41}^n} \qquad n = 1, 2, \ldots, N-1 \tag{110}$$

$$\frac{\partial x_1^N}{\partial \theta_1^N} = \frac{g_{43}^N}{g_{42}^N} \tag{111}$$

$$\frac{\partial x_1^n}{\partial \theta_2^n} = \frac{g_{44}^n}{g_{41}^n} \qquad n = 1, 2, \ldots, N-1 \tag{112}$$

$$\frac{\partial x_1^N}{\partial \theta_2^N} = \frac{g_{44}^N}{g_{42}^N} \tag{113}$$

$$\frac{\partial x_1^n}{\partial \theta_3^n} = \frac{g_{45}^n}{g_{41}^n} \qquad n = 1, 2, \ldots, N-1 \tag{114}$$

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$$\frac{\partial x_1^N}{\partial x_1^{N-1}} = \frac{g_{46}^N}{g_{42}^N} \tag{117}$$

$$\frac{\partial x_1^n}{\partial x_2^{n-1}} = 0 \qquad n = 1, 2, \ldots, N \tag{118}$$

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(2) x_2^n

$$\frac{\partial x_2^n}{\partial \theta_1^n} = h_7^n + h_8^n \qquad n = 1, 2, \ldots, N\text{-}1 \tag{120}$$

$$\frac{\partial x_2^N}{\partial \theta_1^N} = h_7^N + h_8^N + h_{16}^N \tag{121}$$

$$\frac{\partial x_2^n}{\partial \theta_2^n} = h_{17}^n + h_{18}^n + {}_{19}^{n} \qquad n = 1, 2, \ldots, N\text{-}1 \tag{122}$$

$$\frac{\partial x_2^N}{\partial \theta_2^N} = h_{17}^N + h_{18}^N + h_{19}^N + h_{20}^N + h_{21}^N \tag{123}$$

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$$\frac{\partial x_2^N}{\partial \theta_3^N} = h_{22}^N + h_{23}^N + h_{24}^N + h_{25}^N \qquad (125)$$

$$\frac{\partial x_2^n}{\partial x_1^{n-1}} = h_{26}^n \qquad n = 1, 2, \ldots, N-1 \qquad (126)$$

$$\frac{\partial x_2^N}{\partial x_1^{N-1}} = h_{26}^N + h_{27}^N \qquad (127)$$

$$\frac{\partial x_2^n}{\partial x_2^{n-1}} = 1 \qquad n = 1, 2, \ldots, N \qquad (128)$$

$$\frac{\partial x_2^n}{\partial x_3^{n-1}} = h_{29}^n \qquad n = 1, 2, \ldots, N \qquad (129)$$

(3) x_3^n

$$\frac{\partial x_3^n}{\partial \theta_1^n} = 0 \qquad n = 1, 2, \ldots, N \qquad (130)$$

$$\frac{\partial x_3^n}{\partial \theta_2^n} = 1 \qquad n = 1, 2, \ldots, N \qquad (131)$$

$$\frac{\partial x_2^n}{\partial \theta_3^n} = h_{22}^n + h_{23}^n + h_{24}^n \qquad n = 1, 2, \ldots, N-1 \qquad (124)$$

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(3) x_3^n

$$\frac{\partial x_3^n}{\partial \theta_1^n} = 0 \qquad n = 1, 2, \ldots, N \qquad (130)$$

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$$\frac{\partial x_3^n}{\partial \theta_3^n} = 0 \qquad n = 1, 2, \ldots, N \qquad (132)$$

$$\frac{\partial x_3^n}{\partial x_1^{n-1}} = 0 \qquad n = 1, 2, \ldots, N \qquad (133)$$

$$\frac{\partial x_3^n}{\partial x_2^{n-1}} = 0 \qquad n = 1, 2, \ldots, N \qquad (134)$$

$$\frac{\partial x_3^n}{\partial x_3^{n-1}} = 0 \qquad n = 1, 2, \ldots, N \qquad (135)$$

Table 1. Symbol Representation of g_n^n

$$g_1^n = x_1^{n-1} + r^n x_1^n$$

$$g_2^n = \frac{c}{K\theta_2^n \, (\theta_1^n)^{7/8}}$$

$$g_3^n = \frac{b}{K^2(\theta_2^n)^2}$$

$$g_4^n = x_1^n\left[K\theta_2^n \, (1 + r^n) + bg_1\right]$$

$$g_5^n = K\theta_2^n + bx_1^n$$

$$g_6^n = \frac{1}{K \, \theta_2^n} \left(\frac{1 + r^n}{g_1^n} \quad \frac{1}{x_1^n}\right)$$

$$g_7^n = B_1 \, \left(1 - \frac{x_1^o}{x_1^N}\right)$$

$$g_8^n = B_2 \, \left(1 - \frac{x_1^o}{x_1^N}\right)$$

$$g_9^n = (g_2^n - g_3^n) \, \ln \frac{g_4^n}{g_1^n g_5^n} + g_6^n$$

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$$g_{10}^n = g_8^n\theta_3^n x_1^{n-1}$$

Table 1. Symbol Representation of g_n^n (Continued)

$$g_{11}^n = x_1^n g_9^n g_{10}^n$$

$$g_{12}^n = \frac{0.875\ c g_1^n}{K\theta_2^n\ (\theta_1^n)^{15/8}}\ \ln \frac{g_4^n}{g_1^n g_5^n}$$

$$g_{13}^n = -\ \frac{g_1^n(g_2^n - g_3^n)x_1^n}{g_4^n}\ g_5^n g_{10}^n$$

$$g_{14}^n = (g_2^n - g_3^n)\ x_1^n g_{10}^n$$

$$g_{15}^n = \frac{g_{10}^n}{K\theta_2^n g_1^n}\ (1 + r^n)\ x_1^n - g_1^n$$

$$g_{16}^n = r^n g_9^n$$

$$g_{17}^n = \frac{g_1^n\ (g_2^n - g_3^n)}{g_4^n}\ \left(\frac{g_4^n}{x_1^n} + b x_1^n r^n\right)$$

$$g_{18}^n = \frac{-(g_2^n - g_3^n)}{g_5^n}\ (g_5^n r^n + b g_1^n)$$

$$g_{19}^n = \frac{g_1^n}{K\theta_2^n}\left[\frac{1}{(x_1^n)^2} - \frac{(1 + r^n)}{(g_1^n)^2}\ \ r^n\right]$$

$$g_{20}^n = \frac{g_1^n(g_2^n - 2g_3^n)}{\theta_2^n}\ \ln \frac{g_4^n}{g_1^n g_5^n}$$

Table 1. Symbol Representation of g_n^n (Continued)

$$g_{11}^n = x_1^n g_9^n g_{10}^n$$

$$g_{12}^n = \frac{0.875\ c g_1^n}{K\theta_2^n\ (\theta_1^n)^{15/8}}\ \ln \frac{g_4^n}{g_1^n g_5^n}$$

$$g_{13}^n = -\ \frac{g_1^n(g_2^n - g_3^n)x_1^n}{g_4^n}\ g_5^n g_{10}^n$$

$$g_{14}^n = (g_2^n - g_3^n)\ x_1^n g_{10}^n$$

$$g_{15}^n = \frac{g_{10}^n}{K\theta_2^n g_1^n}\ (1 + r^n)\ x_1^n - g_1^n$$

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$$g_{17}^n = \frac{g_1^n\ (g_2^n - g_3^n)}{g_4^n}\ \left(\frac{g_4^n}{x_1^n} + b x_1^n r^n\right)$$

$$g_{18}^n = \frac{-(g_2^n - g_3^n)}{g_5^n}\ (g_5^n r^n + b g_1^n)$$

$$g_{19}^n = \frac{g_1^n}{K\theta_2^n}\left[\frac{1}{(x_1^n)^2} - \frac{(1 + r^n)}{(g_1^n)^2}\ r^n\right]$$

$$g_{20}^n = \frac{g_1^n(g_2^n - 2g_3^n)}{\theta_2^n}\ \ln \frac{g_4^n}{g_1^n g_5^n}$$

Table 1. Symbol Representation of g_n^n (Continued)

$$g_{21}^n = -\frac{g_1^n(g_2^n - g_3^n)}{g_4^n} K x_1^n (1 + r^n)$$

$$g_{22}^n = \frac{K g_1^n(g_2^n - g_3^n)}{g_5}$$

$$g_{23}^n = \frac{g_1^n g_6^n}{\theta_2^n}$$

$$g_{24}^n = g_8^n \theta_1^n x_1^{n-1}$$

$$g_{25}^n = g_7^n x_1^{n-1} - x_1^n g_9^n g_{24}^n$$

$$g_{26}^n = \frac{g_1^n(g_2^n - g_3^n)}{g_4^n} x_1^n g_{24}^n g_5^n$$

$$g_{27}^n = (g_2^n - g_3^n) x_1^n g_{24}^n$$

$$g_{28}^n = -\frac{g_{24}^n}{K\theta_2^n g_1^n}\left[g_1^n - x_1^n (1 + r^n) \right]$$

$$g_{29}^n = g_8^n \theta_1^n \theta_3^n$$

$$g_{30}^n = 1 + x_1^n g_{29}^n$$

$$g_{31}^n = g_7^n \theta_3^n - g_9^n g_{30}^n + (g_2^n - g_3^n) g_{30}^n$$

Table 1. Symbol Representation of g_n^n (Continued)

$$g_{21}^n = - \frac{g_1^n(g_2^n - g_3^n)}{g_4^n} K x_1^n (1 + r^n)$$

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$$g_{23}^n = \frac{g_1^n g_6^n}{\theta_2^n}$$

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$$g_{29}^n = g_8^n \theta_1^n \theta_3^n$$

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Table 1. Symbol Representation of g_n^n (Continued)

$$g_{32}^n = \frac{g_1^n(g_2^n - g_3^n)\, x_1^n}{g_4^n} (K\theta_2^n\, g_{29}^n + b g_{30}^n)$$

$$g_{33}^n = \frac{g_{29}^n g_1^n - (1 + r^n) g_{30}^n}{K\theta_2^n g_1^n}$$

$$g_{34}^n = \theta_3^n\, x_1^{n-1}\, \frac{x_1^o}{(x_1^n)^2}$$

$$g_{35}^n = B_2 \theta_1^n\, g_{34}^n$$

$$g_{36}^n = B_1 g_{34}^n$$

$$g_{37}^n = g_9^n g_{35}^n x_1^n - g_{36}^n$$

$$g_{38}^n = \frac{g_1^n(g_2^n - g_3^n)}{g_4^n}\; x_1^n\, g_5^n\, g_{35}^n$$

$$g_{39}^n = - (g_2^n - g_3^n)\, x_1^n g_{35}^n$$

$$g_{40}^n = \frac{g_{35}^n\, g_1^n - (1 + r^n)\, x_1^n}{K\theta_2^n g_1^n}$$

$$g_{41}^n = g_{16}^n + g_{17}^n + g_{18}^n + g_{19}^n$$

Table 1. Symbol Representation of g_n^n (Continued)

$$g_{32}^n = \frac{g_1^n(g_2^n - g_3^n)\, x_1^n}{g_4^n} (K\theta_2^n\, g_{29}^n + bg_{30}^n)$$

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$$g_{34}^n = \theta_3^n\, x_1^{n-1}\, \frac{x_1^o}{(x_1^n)^2}$$

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Table 1. Symbol Representation of g_n^n (Continued)

$$g_{42}^n = g_{41}^n + g_{37}^n + g_{38}^n + g_{39}^n + g_{40}^n$$

$$g_{43}^n = g_{11}^n + g_{12}^n + g_{13}^n + g_{14}^n + g_{15}^n$$

$$g_{44}^n = g_{20}^n + g_{21}^n + g_{22}^n + g_{23}^n$$

$$g_{45}^n = g_{25}^n + g_{26}^n + g_{27}^n + g_{28}^n$$

$$g_{46}^n = g_{31}^n + g_{32}^n + g_{33}^n$$

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$$g_{45}^n = g_{25}^n + g_{26}^n + g_{27}^n + g_{28}^n$$

$$g_{46}^n = g_{31}^n + g_{32}^n + g_{33}^n$$

Table 2. Symbol Representation of h_n^n

$$h_1^n = \frac{\theta_2^n - \theta_3^{n-1}}{x_1^{n-1}\left(1 - \frac{x_1^o}{x_1^N}\right)}$$

$$h_2^n = (\theta_1^n)^{2.8}\,\theta_3^n$$

$$h_3^n = \theta_3^n\,(\theta_2^n)^{\frac{1}{2}}$$

$$h_4^n = B_8 + B_7\,(\theta_2^n)^{\frac{1}{2}}$$

$$h_5^n = \frac{\theta_2^N}{x_1^N - x_1^o}$$

$$h_6^n = \frac{x_1^N}{x_1^N - x_1^o}$$

$$h_7^n = 2.8a_2B_{13}B_5\theta_3^n\,(\theta_1^n)^{1.8}\,(B_5h_2^n)^{a_2-1}$$

$$h_8^n = 2.8B_{10}\,(\theta_1^n)^{1.8}\,\theta_3^n$$

$$h_9^n = \frac{-x_1^o}{(x_1^n - x_1^o)^2}$$

$$h_{10}^n = \frac{\theta_2^n - x_3^{n-1}}{x_1^{n-1}} \cdot h_9^n$$

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Table 2. Symbol Representation of h_n^n (Continued)

$$h_{11}^n = a_1 B_3 (B_4 h_1^n)^{a_1 - 1} h_{10}^n B_4.$$

$$h_{12}^n = B_9 h_{10}^n$$

$$h_{13}^n = \frac{a_3 B_{14} B_{11} (B_{11} h_5^n)^{a_3 - 1} \theta_2^n h_9^n}{x_1^o}$$

$$h_{14}^n = C_F h_9^n - \frac{B_{12} \theta_2^N h_9^n}{x_1^o}$$

$$h_{15}^n = h_{11}^n + h_{12}^n + h_{13}^n + h_{14}^n$$

$$h_{16}^n = h_{15}^n \frac{\partial x_1^n}{\partial \theta_1^n}$$

$$h_{17}^n = \frac{a_1 B_3 B_4 h_6^n (B_4 h_1^n)^{a_1 - 1}}{x_1^{n-1}}$$

$$h_{18}^n = \frac{a_4 B_6 (h_3^n h_4^n)^{a_4 - 1} (h_4^n + B_7 h_3^n)}{2(\theta_2^n)^{1/2}}$$

$$h_{19}^n = \frac{B_9 h_6^n}{x_1^{n-1}}$$

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Table 2. Symbol Representation of h_n^n (Continued)

$$h_{21}^n = \frac{1}{x_1^N - x_1^o}\left\{a_3 B_{14} B_{11} \left(B_{11} h_5^n\right)^{a_3 - 1} - B_{12}\right\}$$

$$h_{22}^n = a_2 B_{13} \left(B_5 h_2^n\right)^{a_2 - 1} B_5 \left(\theta_1^n\right)^{2.8}$$

$$h_{23}^n = a_4 B_6 \left(h_3^n h_4^n\right)^{a_4 - 1} \left(\theta_2^n\right)^{\frac{1}{2}} h_4^n$$

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5-3. Adjoint Variables

(a) Adjoint Variables z_i^N

Since x_2^N is the total cost function, we have

$$c_1 = 0, \qquad c_2 = 1, \qquad c_3 = 0 \tag{136}$$

and we can write (11)

$$z_1^N = 0, \qquad z_2^N = 1, \qquad z_3^N = 0. \tag{137}$$

However, since x_1^N is prefixed,

$$z_1^N \neq c_1. \tag{138}$$

Then H^N becomes

$$H^N = z_1^N x_1^N + x_2^N$$

Differentiating H^N with respect to θ_3^N yields

$$\frac{\partial H^N}{\partial \theta_3^N} = z_1^N \frac{\partial x_1^N}{\partial \theta_3^N} + \frac{\partial x_2^N}{\partial \theta_3^N}$$

Setting $\dfrac{\partial H^N}{\partial \theta_3^N} = 0$ yields

$$z_1^N = \frac{-\dfrac{\partial x_2^N}{\partial \theta_3^N}}{\dfrac{\partial x_1^N}{\partial \theta_3^N}} \tag{139}$$

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(b) Adjoint Variables z_i^n

From the definition of adjoint variables (11) and the known values of z_i^N and the derivative of state variables in section 5-2, we obtain the following expression for z_i^n

$$z_1^{n-1} = z_i^n \frac{\partial x_1^n}{\partial x_1^{n-1}} + z_2^n \frac{\partial x_2^n}{\partial x_1^{n-1}} \qquad n = 1, 2, \ldots, N \qquad (140)$$

$$z_2^{n-1} = z_1^n \frac{\partial x_1^n}{\partial x_2^{n-1}} + z_2^n \frac{\partial x_2^n}{\partial x_2^{n-1}} = z_1^n \quad n = 1, 2, \ldots, N \qquad (141)$$

$$z_3^{n-1} = z_1^n \frac{\partial x_1^n}{\partial x_3^{n-1}} + z_2^n \frac{\partial x_2^n}{\partial x_3^{n-1}} = z_2^n \frac{\partial x_2^n}{\partial x_3^{n-1}} \quad n = 1, 2, \ldots, N \quad (142)$$

5-4. Derivatives of Hamiltonians

From the definition of the Hamiltonian (11) and the known value of the derivative of the state variables in Section 5-2 and z_i^n in Section 5-3, we have

$$\frac{\partial H^n}{\partial \theta_1^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_1^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_1^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_1^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_1^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_1^n} \qquad n = 1, 2, \ldots, N \qquad (143)$$

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$$z_2^{n-1} = z_1^n \frac{\partial x_1^n}{\partial x_2^{n-1}} + z_2^n \frac{\partial x_2^n}{\partial x_2^{n-1}} = z_2^n \quad n = 1, 2, \ldots, N \qquad (141)$$

$$z_3^{n-1} = z_1^n \frac{\partial x_1^n}{\partial x_3^{n-1}} + z_2^n \frac{\partial x_2^n}{\partial x_3^{n-1}} = z_2^n \frac{\partial x_2^n}{\partial x_3^{n-1}} \quad n = 1, 2, \ldots, N \quad (142)$$

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From the definition of the Hamiltonian (11) and the known value of the derivative of the state variables in Section 5-2 and z_i^n in Section 5-3, we have

$$\frac{\partial H^n}{\partial \theta_1^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_1^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_1^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_1^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_1^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_1^n} \qquad n = 1, 2, \ldots, N \qquad (143)$$

$$\frac{\partial H^n}{\partial \theta_2^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_2^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_2^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_2^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_2^n} + z_3^n \qquad n = 1, 2, \ldots, N \qquad (144)$$

$$\frac{\partial H^n}{\partial \theta_3^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_3^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_3^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_3^n} \qquad n = 1, 2, \ldots, N \qquad (145)$$

5-5. Computing Procedures

A suggested computational procedure to seek the optimal decisions θ_i^n for a fixed x_1^N is as follows:

Step 1. Assume a set of values of θ_1^n (n=1, ..., N) θ_2^n (n=1, ..., N), θ_3^n (n=1, ..., N-1), and $\Delta\theta_i^n$ as a trial.

Step 2. Calculate x_1^n (n=1, ..., N-1) and θ_3^N from equations (103) and (104).

Step 3. Calculate x_2^n (n=1, ..., N-1) and x_2^N from equations (108) and (109).

Step 4. Calculate x_3^n from equation (107)

Step 5. Calculate z_1^N, z_1^{n-1}, z_2^{n-1}, and z_3^{n-1} (n=1, 2, ..., N) from equations (139) and (142).

$$\frac{\partial H^n}{\partial \theta_2^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_2^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_2^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_2^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_2^n} + z_3^n \qquad n = 1, 2, \ldots, N \qquad (144)$$

$$\frac{\partial H^n}{\partial \theta_3^n} = z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_3^n} + z_3^n \frac{\partial x_3^n}{\partial \theta_3^n}$$

$$= z_1^n \frac{\partial x_1^n}{\partial \theta_3^n} + z_2^n \frac{\partial x_2^n}{\partial \theta_3^n} \qquad n = 1, 2, \ldots, N \qquad (145)$$

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Step 4. Calculate x_3^n from equation (107)

Step 5. Calculate z_1^N, z_1^{n-1}, z_2^{n-1}, and z_3^{n-1} (n=1, 2, ..., N) from equations (139) and (142).

Step 6. Calculate $\frac{\partial H^n}{\partial \theta_1^n}$, $\frac{\partial H^n}{\partial \theta_2^n}$, $\frac{\partial H^n}{\partial \theta_3^n}$ from equations (143) through (145).

Step 7. If $\frac{\partial H^n}{\partial \theta_i^n}$ are zero or less than the allowable errors preassigned, then the assumed θ_i^n are the optimal values, otherwise go to the next step.

Step 8. If x_2^N is greater than that computed in the preceding iteration, then one half of the original $\Delta \theta_i^n$ is used; otherwise the original $\Delta \theta_i^n$ is used.

Step 9. The new set of decision (θ_i^n) new is obtained by

$$(\theta_i^n)_{new} = (\theta_i^n)_{old} \pm \Delta \theta_i^n \tag{146}$$

when

$$\frac{\partial H^n}{\partial \theta_i^n} < 0 \qquad \text{use } (-) \text{ sign}$$

when

$$\frac{\partial H^n}{\partial \theta_i^n} > 0 \qquad \text{use } (+) \text{ sign}$$

Then return to step 2 and repeat the computation until the optimum is obtained.

The computational procedure described here is similar to that in Section 5-2 of PART ONE. From the experience in PART ONE, it is believed that the same procedure can be applied to determination of

Step 6. Calculate $\frac{\partial H^n}{\partial \theta_1^n}$, $\frac{\partial H^n}{\partial \theta_2^n}$, $\frac{\partial H^n}{\partial \theta_3^n}$ from equations (143) through (145).

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$$(\theta_i^n)_{new} = (\theta_i^n)_{old} \pm \Delta\theta_i^n \tag{146}$$

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Then return to step 2 and repeat the computation until the optimum is obtained.

The computational procedure described here is similar to that in Section 5-2 of PART ONE. From the experience in PART ONE, it is believed that the same procedure can be applied to determination of

the optimal condition of the process. However, the numerical result is yet to be determined by the actual computer computation.

the optimal condition of the process. However, the numerical result is yet to be determined by the actual computer computation.

NOMENCLATURE

a = a positive exponent of power rule for capital cost of equipment; a_1, a_2, a_3, and a_4 are the exponents for the high pressure pump, the recirculation pump, the turbine, and the membrane separation unit, respectively.

A^n = cross-section area of the membrane separator unit in the n-th stage, ft^2.

c = $3.05 \times 10^5 \dfrac{K \quad d}{(Sc)^{1/3} \; Da}$ constant, ft^3-ft-sec/ft^2-hr-psi-cm^2

C_e = electrical-power cost, \$/psi-ft^3

C_F = the unit cost of brine feed, \$/lb_m

C_t = the total water cost per unit water production, \$/lb_m

C_n = the various cost items (n=1, ..., 7) \$/lb_m

d = the diameter of the membrane tube, ft

D_a = molecular diffusivity of salt, cm^2/sec.

E_1^n = the pumping work of the high pressure pump at the n-th stage for the general model; models A, B, and C are represented respectively by E_{1a}^n, E_{1b}^n, and E_{1c}^n, psi-ft^3/hr.

E_2^n = the pumping work of the recirculation pump at the n-th stage for the general model; models A, B, and C are denoted by E_{1a}^n, E_{2b}^n, and E_{2c}^n, respectively, psi-ft /hr

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E_3 = the energy recovery from the blowdown turbine at the end of the process for the general model; models A, B, and C are denoted by E_{3a}, E_{3b}, and E_{3c}, respectively, psi-ft /hr.

f = Fanning friction factor.

F = the volumetric flux of water through the membrane, ft^3/ft^2-hr.

H^n = Hamiltonian functions of the n-th stage.

J_1^n = the high pressure pump at the n-th stage.

J_2^n = the recirculation pump at the n-th stage.

J_3 = the reject turbine at the last stage.

k = the proportionality constant in the power rule cost expression for the equipment and k_1, k_2, k_t and k_s such proportionality constant for the high pressure pump, the recirculation pump, the turbine, and the membrane separator, respectively.

K = the membrane constant, ft^3/ft^2-hr-psi

L/D = the overall length-to-diameter ratio of the membrane separator.

M^n = the mixing point in the n-th stage.

m^n = the total number of tubes in the membrane separator unit of the n-th stage.

MS^n = the membrane separator unit at the n-th stage

N = the total number of stages in the system

E_3 = the energy recovery from the blowdown turbine at the end of the process for the general model; models A, B, and C are denoted by E_{3a}, E_{3b}, and E_{3c}, respectively, psi-ft /hr.

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P^n = pressure within membrane separator chamber of the n-th stage, psi.

P^o = atmosphere pressure, 14.7 psi.

ΔP^n = pressure difference across the membrane at the n-th stage, psi.

q^n = mass flow rate of brine solution discharged from the n-th stage, lb_m/hr.

q_i^n = mass flow rate the brine entering the membrane separator of the n-th stage, lb_m/hr.

q_e^n = mass flow rate of the brine leaving the membrane separator of the n-th stage, lb_m/hr.

qo = mass flow rate of brine feed, lb_m/hr.

R^n = the flow rate of recycle stream, lb_m/hr.

r^n = the recycle ratio at the n-th stage.

Re^n = Reynolds number at the n-th stage.

S^n = membrane area at the n-th stage, ft^2

Sc = Schmidt number.

T = the capacity of the equipment

W^n = flow of fresh-water produced from the n-th stage, lb_m/hr.

P^n = pressure within membrane separator chamber of the n-th stage, psi.

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T = the capacity of the equipment

W^n = flow of fresh-water produced from the n-th stage, lb_m/hr.

W_f = the total water production from the system, lb_m/hr.

W_s^n = the mass of the shell-and-tube membrane separator unit of the n-th stage for the general model; models A, B, and C are denoted by W_{sa}^n, W_{sb}^n, and W_{sc}^n, respectively, lb_m.

x^n = the mass fraction of salt component in the brine solution leaving the n-th stage.

x_i^n = the mass fraction of salt component in the brine solution entering the membrane separator of the n-th stage.

x_e^n = the mass fraction of salt component in the brine solution leaving the membrane separator of the n-th stage.

x_1^n = x^n

x_2^n = accumulated water cost at the first n stages and $x_2^N = C_t$.

x_3^n = θ_2^n

z_i^n = adjoint variables in association with stage variables x_i^n.

Greek Letters

η_f = loss factor

η_m = mechanical efficiency

η_p = pump efficiency

θ_1^n = R_e^n

W_f = the total water production from the system, lb_m/hr.

W_s^n = the mass of the shell-and-tube membrane separator unit of the n-th stage for the general model; models A, B, and C are denoted by W_{sa}^n, W_{sb}^n, and W_{sc}^n, respectively, lb_m.

x^n = the mass fraction of salt component in the brine solution leaving the n-th stage.

x_i^n = the mass fraction of salt component in the brine solution entering the membrane separator of the n-th stage.

x_e^n = the mass fraction of salt component in the brine solution leaving the membrane separator of the n-th stage.

x_1^n = x^n

x_2^n = accumulated water cost at the first n stages and $x_2^N = C_t$.

x_3^n = θ_2^n

z_i^n = adjoint variables in association with stage variables x_i^n.

Greek Letters

η_f = loss factor

η_m = mechanical efficiency

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θ_1^n = R_e^n

$\theta_2^n = \Delta P^n$

$\theta_3^n = s^n$

μ = viscosity of the brine solution, lbf/in^2.

ρ = density of brine solution, lb_m/ft^3.

ρ_m = density of material of construction, lb_m/ft^3.

σ_m = allowable stress of materials of construction, psi.

ψ = capitalization charge of initial cost per hour in stream, br^{-1}

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APPENDIX

The FORTRAN program symbols, their corresponding mathematical notations and their explanations are summarized in Table A-1. The FORTRAN computer program (1) for the discrete maximum principle is presented in Table A-2; the FORTRAN computer program (2) for the simplex method is given in Table A-3. The input data and sample output results for the computer program (1) are presented in Table A-4.

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Table A-1. PROGRAM SYMBOLS AND EXPLANATION

Program Symbols	Explanation	Mathematic Symbols
A	Steam temperature, 274.4°F	a, or T_s
AR(n)	Condensing area cost in the n-th effect	E_3^n
B	Coefficient of the Clausius-Clapeyron equation including the friction loss, $1.79 \times 10^9 lb_f/ft^2$	B
CC	Cooling water unit cost, 5.9875×10^{-7}\$/lb	C_c
CCD	Unit cost of condensing area, 2.397×10^{-5}\$/ft^2	C_{cd}
CHT	Unit cost of brine heater, 3.76×10^{-5}\$/ft^2	C_{ht}
CP	Heat capacity of water, 1.0Btu/lb,°F	C_p
CPP	Unit pumping cost, 2.903×10^{-9}\$/ft-lb	C_{pp}
CST	Unit steam cost, 2.5×10^{-4}\$/lb	C_{st}
C5	Construction cost, 2.08×10^{-2}\$/1000gal.	E_6
D	Density of water, 62.5 lb/cu.ft.	ρ
DS(n)	Increment of decision variables	$\Delta\delta$
ER	Maximum allowable error for $\frac{H}{-}$	ER
ERROR	Maximum allowable error for standard error function y_s	ERROR
HT	Latent heat of flash brine, 1000Btu/lb	λ
HTS	Latent heat of steam at 274.4°F and 45 psia, 928.9 Btu/lb	λ_s
PC	Unit cost of pretreatment, 1.796×10^{-6}\$/lb	P_c

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B	Coefficient of the Clausius-Clapeyron equation including the friction loss, $1.79\times10^{9}\,lb_f/ft^2$	B
CC	Cooling water unit cost, 5.9875×10^{-7}\$/lb	C_c
CCD	Unit cost of condensing area, 2.397×10^{-5}\$/ft^2	C_{cd}
CHT	Unit cost of brine heater, 3.76×10^{-5}\$/ft^2	C_{ht}
CP	Heat capacity of water, 1.0Btu/lb,°F	C_p
CPP	Unit pumping cost, 2.903×10^{-9}\$/ft-lb	C_{pp}
CST	Unit steam cost, 2.5×10^{-4}\$/lb	C_{st}
C5	Construction cost, 2.08×10^{-2}\$/1000gal.	E_6
D	Density of water, 62.5 lb/cu.ft.	ρ
DS(n)	Increment of decision variables	$\Delta\delta$
ER	Maximum allowable error for $\frac{H}{\ }$	ER
ERROR	Maximum allowable error for standard error function y_s	ERROR
HT	Latent heat of flash brine, 1000Btu/lb	λ
HTS	Latent heat of steam at 274.4°F and 45 psia, 928.9 Btu/lb	λ_s
PC	Unit cost of pretreatment, 1.796×10^{-6}\$/lb	P_c

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
PU(n)	Pump cost and pumping cost in the n-th effect	$E_4^n+E_5^n$
Q	The ratio of heat load to seawater feed	q_s/F
QF(n)	The value of Q at vertex P_n	q_s/F
R	Ideal gas constant, 0.1104 Btu/lb,°R	R
S(n)	Number of stages in the n-th effect	N_n
TE(n)	Average α in the n-th effect	α_n
TEST	Standard error function y_s', $\sqrt{\sum_{i=1}^{3}(y_i-y_4)^2/3}$	y_s'
TV(n)	Direction of decision increment	
T1Y11, H1Y11	Derivative of Hamiltonian H^1 with respect to θ_1^1	$\frac{\partial H^1}{\partial \theta_1^1}$
T1Y31, H1Y31	Derivative of Hamiltonian H^1 with respect to θ_3^1	$\frac{\partial H^1}{\partial \theta_3^1}$
T2Y12, H2Y12	Derivative of Hamiltonian H^2 with respect to θ_1^2	$\frac{\partial H^2}{\partial \theta_1^2}$
T2Y32, H2Y32	Derivative of Hamiltonian H^2 with respect to θ_3^2	$\frac{\partial H^2}{\partial \theta_3^2}$
T3Y13, H3Y13	Derivative of Hamiltonian H^3 with respect to θ_1^3	$\frac{\partial H^3}{\partial \theta_1^3}$
U	Overall heat transfer coefficient, 510 Btu/hr.ft^2.°F	U
V	Unit positive number, 1	1

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
PU(n)	Pump cost and pumping cost in the n-th effect.	$E_4^n+E_5^n$
Q	The ratio of heat load to seawater feed	q_s/F
QF(n)	The value of Q at vertex P_n	q_s/F
R	Ideal gas constant, 0.1104 Btu/lb,°R	R
S(n)	Number of stages in the n-th effect	N_n
TE(n)	Average α in the n-th effect	α_n
TEST	Standard error function y_s', $\sqrt{\sum_{i=1}^{3}(y_i-y_4)^2/3}$	y_s'
TV(n)	Direction of decision increment	
T1Y11, H1Y11	Derivative of Hamiltonian H^1 with respect to θ_1^1	$\frac{\partial H^1}{\partial \theta_1^1}$
T1Y31, H1Y31	Derivative of Hamiltonian H^1 with respect to θ_3^1	$\frac{\partial H^1}{\partial \theta_3^1}$
T2Y12, H2Y12	Derivative of Hamiltonian H^2 with respect to θ_1^2	$\frac{\partial H^2}{\partial \theta_1^2}$
T2Y32, H2Y32	Derivative of Hamiltonian H^2 with respect to θ_3^2	$\frac{\partial H^2}{\partial \theta_3^2}$
T3Y13, H3Y13	Derivative of Hamiltonian H^3 with respect to θ_1^3	$\frac{\partial H^3}{\partial \theta_1^3}$
U	Overall heat transfer coefficient, 510 Btu/hr.ft^2.°F	U
V	Unit positive number, 1	1

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
W	Overall water production rate, 1000 gal./hr. or 8.34×10^3 lb/hr	ΣW_n
WPR(n)	Water production rate in the n-th effect	W_n
WC, X3(4)	Water cost, $/1000gal.	x_2^3
WCI(n)	Function value or water cost at vertex P_n	y_n
X1(n+1)	Outlet concentration in the n-th effect, $(C_f)_n$	x_1^n
XF(n)	$(C_f)_3$ at vertex P_n	x_1^3
DT(n+1)	Temperature gradient in the n-th effect, ΔT^n	ΔT^n
X2(n+1)	Accumulated water production cost in the first n-th effects, $/1000gal.	x_2^n
X3(n+1)	Brine temperature in the n-th effect, $(T_f)_n$	x_3^n
X1Y1(n-1)	Derivative of x_1^n with respect to θ_1^n	$\frac{\partial x_1^n}{\partial \theta_1^n}$
X1Y3(n-1)	Derivative of x_1^n with respect θ_3^n	$\frac{\partial x_1^n}{\partial \theta_3^n}$
DTY1(n-1)	Derivative of ΔT^n with respect to θ_1^n	$\frac{\partial \Delta T^n}{\partial \theta_1^n}$
DTX1(n-2)	Derivative of ΔT^n with respective to x_1^{n-1}	$\frac{\partial \Delta T^n}{\partial x_1^{n-1}}$
X2Y1(n-1)	Derivative of x_2^n with respective to θ_1^n	$\frac{\partial x_2^n}{\partial \theta_1^n}$

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
W	Overall water production rate, 1000 gal./hr. or 8.34×10^3 lb/hr	ΣW_n
WPR(n)	Water production rate in the n-th effect	W_n
WC, X3(4)	Water cost, \$/1000gal.	x_2^3
WCI(n)	Function value or water cost at vertex P_n	y_n
X1(n+1)	Outlet concentration in the n-th effect, $(C_f)_n$	x_1^n
XF(n)	$(C_f)_3$ at vertex P_n	x_1^3
DT(n+1)	Temperature gradient in the n-th effect, ΔT^n	ΔT^n
X2(n+1)	Accumulated water production cost in the first n-th effects, \$/1000gal.	x_2^n
X3(n+1)	Brine temperature in the n-th effect, $(T_f)_n$	x_3^n
X1Y1(n-1)	Derivative of x_1^n with respect to θ_1^n	$\frac{\partial x_1^n}{\partial \theta_1^n}$
X1Y3(n-1)	Derivative of x_1^n with respect θ_3^n	$\frac{\partial x_1^n}{\partial \theta_3^n}$
DTY1(n-1)	Derivative of ΔT^n with respect to θ_1^n	$\frac{\partial \Delta T^n}{\partial \theta_1^n}$
DTX1(n-2)	Derivative of ΔT^n with respective to x_1^{n-1}	$\frac{\partial \Delta T^n}{\partial x_1^{n-1}}$
X2Y1(n-1)	Derivative of x_2^n with respective to θ_1^n	$\frac{\partial x_2^n}{\partial \theta_1^n}$

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
X2Y3(n-1)	Derivative of x_2^n with respect to θ_3^n.	$\frac{\partial x_2^n}{\partial \theta_3^n}$
X2X1(n-2)	Derivative of x_2^n with respect to x_1^{n-1}	$\frac{\partial x_2^n}{\partial x_1^{n-1}}$
X2X3(n-2)	Derivative of x_2^n with respect to x_3^{n-1}	$\frac{\partial x_2^n}{\partial x_3^{n-1}}$
TA	Coefficient of reflection	α'
TB	Coefficient of contraction	β'
TR	Coefficient of expansion	γ'
Y1(n)	Recycle ratio in the n-th effect, γ_n	θ_1^n
Y3(n)	Temperature drop in the n-th effect	$\theta_2^n - \theta_2^{n-1}$
Z11	Adjoint variable, z_1^1	z_1^1
Z12	Adjoint variable, z_1^2	z_1^2
Z13	Adjoint variable, z_1^3	z_1^3
Z31	Adjoint variable, z_3^1.	z_3^1
Z32	Adjoint variable, z_3^2	z_3^2

Table A-1 (Continued)

Program Symbols	Explanation	Mathematic Symbols
X2Y3(n-1)	Derivative of x_2^n with respect to θ_3^n	$\frac{\partial x_2^n}{\partial \theta_3^n}$
X2X1(n-2)	Derivative of x_2^n with respect to x_1^{n-1}	$\frac{\partial x_2^n}{\partial x_1^{n-1}}$
X2X3(n-2)	Derivative of x_2^n with respect to x_3^{n-1}	$\frac{\partial x_2^n}{\partial x_3^{n-1}}$
TA	Coefficient of reflection	α'
TB	Coefficient of contraction	β'
TR	Coefficient of expansion	γ'
Y1(n)	Recycle ratio in the n-th effect, γ_n	θ_1^n
Y3(n)	Temperature drop in the n-th effect	$\theta_2^n - \theta_2^{n-1}$
Z11	Adjoint variable, z_1^1	z_1^1
Z12	Adjoint variable, z_1^2	z_1^2
Z13	Adjoint variable, z_1^3	z_1^3
Z31	Adjoint variable, z_3^1	z_3^1
Z32	Adjoint variable, z_3^2	z_3^2

Table A-2 Computer Program (1)

```
   FLASH DISTILLATION BY MAXIMUM PRINCIPLE     BY KIANG KOH-DON
   DIMENSION Y1(4),Y3(4),X1(4),X2(4),X3(4),DT(4),X1Y1(3),S(4)
   DIMENSION X1Y3(3),DTY1(3),X2Y1(3),X2Y3(3),DTX1(2),X2X3(2),X2X1(2)
   DIMENSION CPR(4),DS(5),TV(5),PU(4),AR(4),TE(3)
 1 FORMAT(10F7.1)
 2 FORMAT(7E10.3)
 3 FORMAT(5E12.5)
 4 FORMAT(6E12.5)
 5 FORMAT(2X,37HTHE FOLLOWING ARE OPTIMUN OUTPUT DATA,/)
 6 FORMAT(2X,6HY1(2)=E12.5,7X,6HY1(3)=E12.5,7X,6HY1(4)=E12.5)
 7 FORMAT(2X,6HY3(2)=E12.5,7X,6HY3(3)=E12.5,7X,6HY3(4)=E12.5)
 8 FORMAT(2X,6HX1(1)=E12.5,7X,6HX1(2)=E12.5,7X,6HX1(3)=E12.5)
 9 FORMAT(2X,6HDT(2)=E12.5,7X,6HDT(3)=E12.5,7X,6HDT(4)=E12.5)
10 FORMAT(2X,6HX3(1)=E12.5,7X,6HX3(2)=E12.5,7X,6HX3(3)=E12.5)
11 FORMAT(2X,6HX2(2)=E12.5,7X,6HX2(3)=E12.5,7X,6HX2(4)=E12.5)
12 FORMAT(2X,6HH1Y11=E12.5,7X,6HH2Y12=E12.5,7X,6HH3Y13=E12.5)
13 FORMAT(2X,6HH1Y31=E12.5,7X,6HH2Y32=E12.5,11X,2HW=E12.5,/)
14 FORMAT(2X,5HTERMS,20X,7HCOST(S),7X,10HPERCENTAGE,/)
15 FORMAT(2X,5HSTEAM,15X,E12.5,5X,E12.5)
16 FORMAT(2X,6HHEATER,14X,E12.5,5X,E12.5)
17 FORMAT(2X,15HCONDENSING AREA,5X,E12.5,5X,E12.5)
18 FORMAT(2X,7HPUMPING,13X,E12.5,5X,E12.5)
19 FORMAT(2X,12HCONSTRUCTION,8X,E12.5,5X,E12.5)
20 FORMAT(2X,12HPRETREATMENT,8X,E12.5,5X,E12.5,/)
21 FORMAT(2X,5HTERMS,12X,11H1 ST EFFECT,6X,11H2 ND EFFECT,6X,11H3 RD
  1EFFECT,/)
22 FORMAT(2X,10HTEMP. DROP,6X,E12.5,5X,E12.5,5X,E12.5)
23 FORMAT(2X,11HWATER PROD.,5X,E12.5,5X,E12.5,5X,E12.5,/)
24 FORMAT(2X,26HREAD NEW OPTIMIZATION DATA)
25 FORMAT(2X,6HX1(4)=E12.5,11X,2HQ=E12.5,7X,6HX2(4)=E12.5)
26 FORMAT(2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,/)
27 FORMAT(2X,10HCOND. COST,6X,E12.5,5X,E12.5,5X,E12.5)
28 FORMAT(2X,10HPUMP. COST,6X,E12.5,5X,E12.5,5X,E12.5)
29 FORMAT(2X,8HS.P. EL.,8X,E12.5,5X,E12.5,5X,E12.5,/)
40 READ(1,4)X1(4),Q,WC,TE(1),TE(2),TE(3)
   READ(1,3)Y1(2),Y1(3),Y1(4),Y3(2),Y3(3)
   READ(1,4)DS(1),DS(2),DS(3),DS(4),DS(5),ER
   READ(1,1)U,A,HT,HTS,X3(1),D,S(2),S(3),S(4),V
   READ(1,2)R,X1(1),CST,W,CHT,CCD,CPP
   READ(1,3)B,CS,PC,CP,CC
71 N=1
   DO 76 I=1,5
76 TV(I)=1.0
41 DO 42 I=2,3
   X=EXP(-CP*Y3(I)/HT)
42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
```

Table A-2 Computer Program (1)

```
      FLASH DISTILLATION BY MAXIMUM PRINCIPLE     BY KIANG KOH-DON
      DIMENSION Y1(4),Y3(4),X1(4),X2(4),X3(4),DT(4),X1Y1(3),S(4)
      DIMENSION X1Y3(3),DTY1(3),X2Y1(3),X2Y3(3),DTX1(2),X2X3(2),X2X1(2)
      DIMENSION CPR(4),DS(5),TV(5),PU(4),AR(4),TE(3)
    1 FORMAT(10F7.1)
    2 FORMAT(7E10.3)
    3 FORMAT(5E12.5)
    4 FORMAT(6E12.5)
    5 FORMAT(2X,37HTHE FOLLOWING ARE OPTIMUN OUTPUT DATA,/)
    6 FORMAT(2X,6HY1(2)=E12.5,7X,6HY1(3)=E12.5,7X,6HY1(4)=E12.5)
    7 FORMAT(2X,6HY3(2)=E12.5,7X,6HY3(3)=E12.5,7X,6HY3(4)=E12.5)
    8 FORMAT(2X,6HX1(1)=E12.5,7X,6HX1(2)=E12.5,7X,6HX1(3)=E12.5)
    9 FORMAT(2X,6HDT(2)=E12.5,7X,6HDT(3)=E12.5,7X,6HDT(4)=E12.5)
   10 FORMAT(2X,6HX3(1)=E12.5,7X,6HX3(2)=E12.5,7X,6HX3(3)=E12.5)
   11 FORMAT(2X,6HX2(2)=E12.5,7X,6HX2(3)=E12.5,7X,6HX2(4)=E12.5)
   12 FORMAT(2X,6HH1Y11=E12.5,7X,6HH2Y12=E12.5,7X,6HH3Y13=E12.5)
   13 FORMAT(2X,6HH1Y31=E12.5,7X,6HH2Y32=E12.5,11X,2HW=E12.5,/)
   14 FORMAT(2X,5HTERMS,20X,7HCOST(S),7X,10HPERCENTAGE,/)
   15 FORMAT(2X,5HSTEAM,15X,E12.5,5X,E12.5)
   16 FORMAT(2X,6HHEATER,14X,E12.5,5X,E12.5)
   17 FORMAT(2X,15HCONDENSING AREA,5X,E12.5,5X,E12.5)
   18 FORMAT(2X,7HPUMPING,13X,E12.5,5X,E12.5)
   19 FORMAT(2X,12HCONSTRUCTION,8X,E12.5,5X,E12.5)
   20 FORMAT(2X,12HPRETREATMENT,8X,E12.5,5X,E12.5,/)
   21 FORMAT(2X,5HTERMS,12X,11H1 ST EFFECT,6X,11H2 ND EFFECT,6X,11H3 RD
     1EFFECT,/)
   22 FORMAT(2X,10HTEMP. DROP,6X,E12.5,5X,E12.5,5X,E12.5)
   23 FORMAT(2X,11HWATER PROD.,5X,E12.5,5X,E12.5,5X,E12.5,/)
   24 FORMAT(2X,26HREAD NEW OPTIMIZATION DATA)
   25 FORMAT(2X,6HX1(4)=E12.5,11X,2HQ=E12.5,7X,6HX2(4)=E12.5)
   26 FORMAT(2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,/)
   27 FORMAT(2X,10HCOND. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   28 FORMAT(2X,10HPUMP. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   29 FORMAT(2X,8HS.P. EL.,8X,E12.5,5X,E12.5,5X,E12.5,/)
   40 READ(1,4)X1(4),Q,WC,TE(1),TE(2),TE(3)
      READ(1,3)Y1(2),Y1(3),Y1(4),Y3(2),Y3(3)
      READ(1,4)DS(1),DS(2),DS(3),DS(4),DS(5),ER
      READ(1,1)U,A,HT,HTS,X3(1),D,S(2),S(3),S(4),V
      READ(1,2)R,X1(1),CST,W,CHT,CCD,CPP
      READ(1,3)B,CS,PC,CP,CC
   71 N=1
      DO 70 I=1,5
   70 TV(I)=1.0
   41 DO 42 I=2,3
      X=EXP(-CP*Y3(I)/HT)
   42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
```

Table A-2 (Continued)

```
42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
   Y3(4)=HT*ALOG((X1(3)+Y1(4)*X1(4))/(X1(4)*(1.+Y1(4))))/CP
   DO 43 I=2,4
   X3(I)=X3(I-1)+Y3(I)
43 DT(I)=(Q/CP+TE(I-1)*(1.-X1(1)/X1(I-1)))/(1.+Y1(I)*X1(1)/X1(I-1))
   WA=W/(1.-X1(1)/X1(4))
   C2=CHT*Q*WA/(U*(A-X3(1)+0.5*DT(2)))
   C1=CST*Q*WA/HTS
   X2(1)=C2+C5+C1
   DO 44 I=2,4
   WPR(I-1)=WA*X1(1)*(1./X1(I-1)-1./X1(I))
   X=WPR(I-1)*HT/U
   AR(I)=CCD*X/(DT(I)-TE(I-1)+Y3(I)/(2.*S(I)))
   Y=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
   PU(I)=CPP*X1(1)*Y1(I)*B*Y*WA/(D*X1(I-1))
   X2(I)=X2(I-1)+AR(I)+PU(I)
   X1Y1(I-1)=X1(I)*(X1(I)-X1(I-1))/(X1(I-1)*(1.+Y1(I)))
   X1Y3(I-1)=-CP*X1(I)*(X1(I-1)+Y1(I)*X1(I))/(HT*X1(I-1))
   DTY1(I-1)=-DT(I)/(X1(I-1)/X1(I)+Y1(I))
   XB1=X1(1)*X1Y1(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
   XB3=X1(1)*DTY1(I-1)*(1./X1(I-1)-1./X1(I))
   XB2=XB3/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
   DT(1)=CCD*HT*WA*(XB1-XB2)/U
   YA1=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
   Y1(1)=CPP*X1(1)*B*YA1*WA/(D*X1(I-1))
   X2Y1(I-1)=DT(1)+Y1(1)
   XB1=X1(1)*X1Y3(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
   XB3=X1(1)*(1./X1(I-1)-1./X1(I))
   XB2=XB3/(2.0*S(I)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
   XA=HT*EXP(-HT/(R*(X3(I-1)+Y3(I))))/(R*(X3(I-1)+Y3(I))**2)
   Y=-CPP*X1(1)*Y1(I)*B*WA*XA/(X1(I-1)*D)
44 X2Y3(I-1)=CCD*HT*WA*(XB1-XB2)/U+Y
   BT=X3(3)+Y3(4)-545.
   CT=(G*WA+TE(3)*W)/BT
   C6=(PC-CC)*WA+CC*CT
   X2(4)=X2(4)+C6
   AR(1)=AR(2)+AR(3)+AR(4)
   PU(1)=PU(2)+PU(3)+PU(4)
   WPR(4)=WPR(1)+WPR(2)+WPR(3)
   PC1=C1*100./X2(4)
   PC2=C2*100./X2(4)
   PC3=AR(1)*100./X2(4)
   PC4=PU(1)*100./X2(4)
   PC5=C5*100./X2(4)
   PC6=C6*100./X2(4)
```

Table A-2 (Continued)

```
42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
   Y3(4)=HT*ALOG((X1(3)+Y1(4)*X1(4))/(X1(4)*(1.+Y1(4))))/CP
   DO 43 I=2,4
   X3(I)=X3(I-1)+Y3(I)
43 DT(I)=(Q/CP+TE(I-1)*(1.-X1(1)/X1(I-1)))/(1.+Y1(I)*X1(1)/X1(I-1))
   WA=W/(1.-X1(1)/X1(4))
   C2=CHT*Q*WA/(U*(A-X3(1)+0.5*DT(2)))
   C1=CST*Q*WA/HTS
   X2(1)=C2+C5+C1
   DO 44 I=2,4
   WPR(I-1)=WA*X1(1)*(1./X1(I-1)-1./X1(I))
   X=WPR(I-1)*HT/U
   AR(I)=CCD*X/(DT(I)-TE(I-1)+Y3(I)/(2.*S(I)))
   Y=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
   PU(I)=CPP*X1(1)*Y1(I)*B*Y*WA/(D*X1(I-1))
   X2(I)=X2(I-1)+AR(I)+PU(I)
   X1Y1(I-1)=X1(I)*(X1(I)-X1(I-1))/(X1(I-1)*(1.+Y1(I))).
   X1Y3(I-1)=-CP*X1(I)*(X1(I-1)+Y1(I)*X1(I))/(HT*X1(I-1))
   DTY1(I-1)=-DT(I)/(X1(I-1)/X1(1)+Y1(I))
   XB1=X1(1)*X1Y1(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
   XB3=X1(1)*DTY1(I-1)*(1./X1(I-1)-1./X1(I))
   XB2=XB3/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
   DT(1)=CCD*HT*WA*(XB1-XB2)/U
   YA1=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
   Y1(1)=CPP*X1(1)*B*YA1*WA/(D*X1(I-1))
   X2Y1(I-1)=DT(1)+Y1(1)
   XB1=X1(1)*X1Y3(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
   XB3=X1(1)*(1./X1(I-1)-1./X1(I))
   XB2=XB3/(2.0*S(I)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
   XA=HT*EXP(-HT/(R*(X3(I-1)+Y3(I))))/(R*(X3(I-1)+Y3(I))**2)
   Y=-CPP*X1(1)*Y1(I)*B*WA*XA/(X1(I-1)*D)
44 X2Y3(I-1)=CCD*HT*WA*(XB1-XB2)/U+Y
   BT=X3(3)+Y3(4)-545.
   CT=(G*WA+TE(3)*W)/BT
   C6=(PC-CC)*WA+CC*CT
   X2(4)=X2(4)+C6
   AR(1)=AR(2)+AR(3)+AR(4)
   PU(1)=PU(2)+PU(3)+PU(4)
   WPR(4)=WPR(1)+WPR(2)+WPR(3)
   PC1=C1*100./X2(4)
   PC2=C2*100./X2(4)
   PC3=AR(1)*100./X2(4)
   PC4=PU(1)*100./X2(4)
   PC5=C5*100./X2(4)
   PC6=C6*100./X2(4)
```

Table A-2 (Continued)

```
      TD1=X3(1)-X3(2)
      TD2=X3(2)-X3(3)
      TD3=X3(3)-X3(4)
      XB1=-CCD*W*X1(1)*X1Y1(3)*V*HT*X1(1)*(1./X1(3)-1./X1(4))/U
      XB2=XB1/(((X1(4)-X1(1))**2)*(DT(4)-TE(3)+0.5*Y3(4)/S(4)))
      X5A=-CPP*X1(1)*Y1(4)*B*X1(1)*X1Y1(3)*W/((X1(4)-X1(1))**2)
      X2Y1(3)=X2Y1(3)+XB2+X5A*YA1/(D*X1(3))
      PT=-WA*X1(1)/(X1(4)*(X1(4)-X1(1)))
      AT=PT*X1Y1(3)
      X2Y1(3)=X2Y1(3)+(PC-CC)*AT+CC*Q*AT/BT
      X2Y1(1)=X2Y1(1)-CHT*Q*WA*DTY1(1)/(U*TE(1)*(A-X3(1)+0.5*DT(2))**2)
      XB1=X1(1)*(1.0/X1(3)-1.0/X1(4))*HT/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=-(W*X1(1)*X1Y3(3)*V/((X1(4)-X1(1))**2))*XB1*CCD/U
      X=W*X1(1)*X1Y3(3)*YA1*B*X1(1)*Y1(4)/((X1(4)-X1(1))**2)
      X2Y3(3)=X2Y3(3)+XB2-CPP*X/(D*X1(3))
      X2Y3(3)=X2Y3(3)+(PC-CC)*PT*X1Y3(3)+CC*(Q*PT*X1Y3(3)/BT-CT/BT)
      DO 45 I=3,4
      K=I-2
      XQ=1.0*X1(1)*(TE(I-1)+X2(I)*Y1(I))
      DTX1(K)=XQ/(X1(I-1)*(X1(I-1)+Y1(I)*X1(1)))
      X=(-1./(X1(I-1)**2)+1./(X1(I-1)*X1(I)))
      X=X/(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))
      YA=DTX1(K)*(1./X1(I-1)-1./X1(I))
      YA=YA/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      XA=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
      Y=-CPP*X1(1)*Y1(I)*B*XA*WA/(D*X1(I-1)**2)
      X2X1(K)=Y+CCD*HT*WA*X1(1)*(X-YA)/U
      X5=EXP(-HT/(R*X3(I-1)))/(X3(I-1)**2)
      YA=EXP(-HT/(R*(X3(I-1)+Y3(I))))/((X3(I-1)+Y3(I))**2)
45    X2X3(K)=1.0*CPP*X1(1)*Y1(I)*WA*B*HT*(X5-YA)/(R*D*X1(I-1))
      XB1=X1(1)*HT*(1.0/X1(3)-1.0/X1(4))/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=W*X1(1)*X1(4)/(X1(3)*(X1(4)-X1(1))**2)
      Y=-CPP*X1(1)*Y1(4)*B*XB2*XA/(D*X1(3))
      X2X1(2)=X2X1(2)+Y-CCD*XB2*XB1/U
      X2X1(2)=X2X1(2)+(PC-CC)*PT*X1(4)/X1(3)+CC*Q*PT*X1(4)/(X1(3)*BT)
      X2X3(2)=X2X3(2)-CC*CT/BT
      Z13=-X2Y3(3)/X1Y3(3)
      Z12=Z13*X1(4)/X1(3)+X2X1(2)
      Z32=X2X3(2)
      Z11=Z12*X1(3)/X1(2)+X2X1(1)
      Z31=X2X3(1)+Z32
      A1Y11=Z11*X1Y1(1)+X2Y1(1)
      A1Y31=Z11*X1Y3(1)+X2Y3(1)+Z31
      A2Y12=Z12*X1Y1(2)+X2Y1(2)
      A2Y32=Z12*X1Y3(2)+X2Y3(2)+Z32
```

Table A-2 (Continued)

```
TD1=X3(1)-X3(2)
TD2=X3(2)-X3(3)
TD3=X3(3)-X3(4)
XB1=-CCD*W*X1(1)*X1Y1(3)*V*HT*X1(1)*(1./X1(3)-1./X1(4))/U
XB2=XB1/(((X1(4)-X1(1))**2)*(DT(4)-TE(3)+0.5*Y3(4)/S(4)))
X5A=-CPP*X1(1)*Y1(4)*B*X1(1)*X1Y1(3)*W/((X1(4)-X1(1))**2)
X2Y1(3)=X2Y1(3)+XB2+X5A*YA1/(D*X1(3))
PT=-WA*X1(1)/(X1(4)*(X1(4)-X1(1)))
AT=PT*X1Y1(3)
X2Y1(3)=X2Y1(3)+(PC-CC)*AT+CC*Q*AT/BT
X2Y1(1)=X2Y1(1)-CHT*Q*WA*DTY1(1)/(U*TE(1)*(A-X3(1)+0.5*DT(2))**2)
XB1=X1(1)*(1.0/X1(3)-1.0/X1(4))*HT/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
XB2=-(W*X1(1)*X1Y3(3)*V/((X1(4)-X1(1))**2))*XB1*CCD/U
X=W*X1(1)*X1Y3(3)*YA1*B*X1(1)*Y1(4)/((X1(4)-X1(1))**2)
X2Y3(3)=X2Y3(3)+XB2-CPP*X/(D*X1(3))
X2Y3(3)=X2Y3(3)+(PC-CC)*PT*X1Y3(3)+CC*(Q*PT*X1Y3(3)/BT-CT/BT)
DO 45 I=3,4
K=I-2
XQ=1.0*X1(1)*(TE(I-1)+X2(I)*Y1(I))
DTX1(K)=XQ/(X1(I-1)*(X1(I-1)+Y1(I)*X1(1)))
X=(-1./(X1(I-1)**2)+1./(X1(I-1)*X1(I)))
X=X/(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))
YA=DTX1(K)*(1./X1(I-1)-1./X1(I))
YA=YA/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
XA=EXP(-HT/(R*X3(I-1)))-EXP(-HT/(R*(X3(I-1)+Y3(I))))
Y=-CPP*X1(1)*Y1(I)*B*XA*WA/(D*X1(I-1)**2)
X2X1(K)=Y+CCD*HT*WA*X1(1)*(X-YA)/U
X5=EXP(-HT/(R*X3(I-1)))/(X3(I-1)**2)
YA=EXP(-HT/(R*(X3(I-1)+Y3(I))))/((X3(I-1)+Y3(I))**2)
45 X2X3(K)=1.0*CPP*X1(1)*Y1(I)*WA*B*HT*(X5-YA)/(R*D*X1(I-1))
XB1=X1(1)*HT*(1.0/X1(3)-1.0/X1(4))/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
XB2=W*X1(1)*X1(4)/(X1(3)*(X1(4)-X1(1))**2)
Y=-CPP*X1(1)*Y1(4)*B*XB2*XA/(D*X1(3))
X2X1(2)=X2X1(2)+Y-CCD*XB2*XB1/U
X2X1(2)=X2X1(2)+(PC-CC)*PT*X1(4)/X1(3)+CC*Q*PT*X1(4)/(X1(3)*BT)
X2X3(2)=X2X3(2)-CC*CT/BT
Z13=-X2Y3(3)/X1Y3(3)
Z12=Z13*X1(4)/X1(3)+X2X1(2)
Z32=X2X3(2)
Z11=Z12*X1(3)/X1(2)+X2X1(1)
Z31=X2X3(1)+Z32
A1Y11=Z11*X1Y1(1)+X2Y1(1)
A1Y31=Z11*X1Y3(1)+X2Y3(1)+Z31
A2Y12=Z12*X1Y1(2)+X2Y1(2)
A2Y32=Z12*X1Y3(2)+X2Y3(2)+Z32
```

Table A-2 (Continued)

```
      H3Y13=Z13*X1Y1(3)+X2Y1(3)
      IF(M-1)66,66,67
   66 TE(1)=1.01+((X1(1)+Y1(2)*X1(2))/(1.+Y1(2))+X1(2))/0.06
      TE(2)=1.0075+((X1(2)+Y1(3)*X1(3))/(1.+Y1(3))+X1(3))/0.0694
      TE(3)=0.32+((X1(3)+Y1(4)*X1(4))/(1.+Y1(4))+X1(4))/0.0630
      M=M+1
      GO TO 41
   67 IF(ABS(H1Y11)-ER)47,47,51
   47 IF(ABS(H1Y31)-ER)48,48,51
   48 IF(ABS(H2Y12)-ER)49,49,51
   49 IF(ABS(H2Y32)-ER)50,50,51
   50 IF(ABS(H3Y13)-ER)63,63,51
   51 IF (H1Y11) 137,137,138
  137 TV(1)=-1.
  138 IF (H2Y12) 139,139,140
  139 TV(2)=-1.
  140 IF (H3Y13) 141,141,142
  141 TV(3)=-1.
  142 IF (H1Y31) 143,143,144
  143 TV(4)=-1.
  144 IF (H2Y32) 145,145,146
  145 TV(5)=-1.
  146 IF(WC-X2(4))36,36,35
   36 DO 37 I=1,5
   37 DS(I)=0.5*DS(I)
      Y1(2)=Y11T
      Y1(3)=Y12T
      Y1(4)=Y13T
      Y3(2)=Y31T
      Y3(3)=Y32T
      GO TO 38
   35 WC=X2(4)
   38 Y11T=Y1(2)
      Y12T=Y1(3)
      Y13T=Y1(4)
      Y31T=Y3(2)
      Y32T=Y3(3)
      Y1(2)=Y1(2)-TV(1)*DS(1)
      Y1(3)=Y1(3)-TV(2)*DS(2)
      Y1(4)=Y1(4)-TV(3)*DS(3)
      Y3(2)=Y3(2)-TV(4)*DS(4)
      Y3(3)=Y3(3)-TV(5)*DS(5)
      GO TO 71
   63 WRITE(3,5)
   65 WRITE(3,25)X1(4),0,X2(4)
```

Table A-2 (Continued)

```
      H3Y13=Z13*X1Y1(3)+X2Y1(3)
      IF(M-1)66,66,67
   66 TE(1)=1.01+((X1(1)+Y1(2)*X1(2))/(1.+Y1(2))+X1(2))/0.06
      TE(2)=1.0075+((X1(2)+Y1(3)*X1(3))/(1.+Y1(3))+X1(3))/0.0694
      TE(3)=0.32+((X1(3)+Y1(4)*X1(4))/(1.+Y1(4))+X1(4))/0.0630
      M=M+1
      GO TO 41
   67 IF(ABS(H1Y11)-ER)47,47,51
   47 IF(ABS(H1Y31)-ER)48,48,51
   48 IF(ABS(H2Y12)-ER)49,49,51
   49 IF(ABS(H2Y32)-ER)50,50,51
   50 IF(ABS(H3Y13)-ER)63,63,51
   51 IF (H1Y11) 137,137,138
  137 TV(1)=-1.
  138 IF (H2Y12) 139,139,140
  139 TV(2)=-1.
  140 IF (H3Y13) 141,141,142
  141 TV(3)=-1.
  142 IF (H1Y31) 143,143,144
  143 TV(4)=-1.
  144 IF (H2Y32) 145,145,146
  145 TV(5)=-1.
  146 IF(WC-X2(4))36,36,35
   36 DO 37 I=1,5
   37 DS(I)=0.5*DS(I)
      Y1(2)=Y11T
      Y1(3)=Y12T
      Y1(4)=Y13T
      Y3(2)=Y31T
      Y3(3)=Y32T
      GO TO 38
   35 WC=X2(4)
   38 Y11T=Y1(2)
      Y12T=Y1(3)
      Y13T=Y1(4)
      Y31T=Y3(2)
      Y32T=Y3(3)
      Y1(2)=Y1(2)-TV(1)*DS(1)
      Y1(3)=Y1(3)-TV(2)*DS(2)
      Y1(4)=Y1(4)-TV(3)*DS(3)
      Y3(2)=Y3(2)-TV(4)*DS(4)
      Y3(3)=Y3(3)-TV(5)*DS(5)
      GO TO 71
   63 WRITE(3,5)
   65 WRITE(3,25)X1(4),Q,X2(4)
```

Table A-2 (Continued)

```
      WRITE(3,6)Y1(2),Y1(3),Y1(4)
      WRITE(3,7)Y3(2),Y3(3),Y3(4)
      WRITE(3,8)X1(1),X1(2),X1(3)
      WRITE(3,9)DT(2),DT(3),DT(4)
      DO 64 I=2,4
   64 X3(I)=X3(I)-460.
      WRITE(3,10)X3(2),X3(3),X3(4)
      WRITE(3,11)X2(1),X2(2),X2(3)
      WRITE(3,12)H1Y11,H2Y12,H3Y13
      WRITE(3,13)H1Y31,H2Y3,WPR(4)
      WRITE(3,26)DS(1),DS(2),DS(3),DS(4),DS(5)
      WRITE(3,14)
      WRITE(3,15)C1,PC1
      WRITE(3,16)C2,PC2
      WRITE(3,17)AR(1),PC3
      WRITE(3,18)PU(1),PC4
      WRITE(3,19)C5,PC5
      WRITE(3,20)C6,PC6
      WRITE(3,21)
      WRITE(3,22)TD1,TD2,TD3
      WRITE(3,23)WPR(1),WPR(2),WPR(3)
      WRITE(3,27)AR(2),AR(3),AR(4)
      WRITE(3,28)PU(2),PU(3),PU(4)
      WRITE(3,29)TE(1),TE(2),TE(3)
      WRITE(3,24)
      GO TO 40
      END
```

Table A-2 (Continued)

```
      WRITE(3,6)Y1(2),Y1(3),Y1(4)
      WRITE(3,7)Y3(2),Y3(3),Y3(4)
      WRITE(3,8)X1(1),X1(2),X1(3)
      WRITE(3,9)DT(2),DT(3),DT(4)
      DO 64 I=2,4
   64 X3(I)=X3(I)-460.
      WRITE(3,10)X3(2),X3(3),X3(4)
      WRITE(3,11)X2(1),X2(2),X2(3)
      WRITE(3,12)H1Y11,H2Y12,H3Y13
      WRITE(3,13)H1Y31,H2Y3,WPR(4)
      WRITE(3,26)DS(1),DS(2),DS(3),DS(4),DS(5)
      WRITE(3,14)
      WRITE(3,15)C1,PC1
      WRITE(3,16)C2,PC2
      WRITE(3,17)AR(1),PC3
      WRITE(3,18)PU(1),PC4
      WRITE(3,19)C5,PC5
      WRITE(3,20)C6,PC6
      WRITE(3,21)
      WRITE(3,22)TD1,TD2,TD3
      WRITE(3,23)WPR(1),WPR(2),WPR(3)
      WRITE(3,27)AR(2),AR(3),AR(4)
      WRITE(3,28)PU(2),PU(3),PU(4)
      WRITE(3,29)TE(1),TE(2),TE(3)
      WRITE(3,24)
      GO TO 40
      END
```

Table A-3 Computer Program (2)

```
      FLASH DISTILLATION BY SIMPLEX METHOD AND MAXIMUM PRINCIPLE
      DIMENSION Y1(4),Y3(4),X1(4),X2(4),X3(4),DT(4),X1Y1(3),S(4)
      DIMENSION X1Y3(3),X2Y1(3),DTY1(3),XIY3(3),DTX1(2),X2X3(2),X2X1(2)
      DIMENSION WPR(4),DS(5),TV(5),PU,4),AR(4),TE(3),QF(6),XF(6),WCI(6)
      DIMENSION QF(6),XF(6),WCI(6)
    1 FORMAT(10F7.1)
    2 FORMAT(7E10.3)
    3 FORMAT(5E12.5)
    4 FORMAT(6E12.5)
    5 FORMAT(2X,37HTHE FOLLOWING ARE OPTIMUN OUTPUT DATA,/)
    6 FORMAT(2X,6HY1(2)=E12.5,7X,6HY1(3)=E12.5,7X,6HY1(4)=E12.5)
    7 FORMAT(2X,6HY3(2)=E12.5,7X,6HY3(3)=E12.5,7X,6HY3(4)=E12.5)
    8 FORMAT(2X,6HX1(1)=E12.5,7X,6HX1(2)=E12.5,7X,6HX1(3)=E12.5)
    9 FORMAT(2X,6HDT(2)=E12.5,7X,6HDT(3)=E12.5,7X,6HDT(4)=E12.5)
   10 FORMAT(2X,6HX2(2)=E12.5,7X,6HX2(3)=E12.5,7X,6HX2(4)=E12.5)
   11 FORMAT(2X,6HX3(1)=E12.5,7X,6HX3(2)=E12.5,7X,6HX3(3)=E12.5)
   12 FORMAT(2X,6HH1Y11=E12.5,7X,6HH2Y12=E12.5,7X,6HH3Y13=E12.5)
   13 FORMAT(2X,6HH1Y31=E12.5,7X,6HH2Y32=E12.5,7X,2HW=E12.5,/)
   14 FORMAT(2X,5HTERMS,20X,7HCOST($),7X,10HPERCENTAGE,/)
   15 FORMAT(2X,5HSTEAM,15X,E12.5,5X,E12.5)
   16 FORMAT(2X,6HHEATER,14X,E12.5,5X,E12.5)
   17 FORMAT(2X,15HCONDENSING AREA,5X,E12.5,5X,E12.5)
   18 FORMAT(2X,7HPUMPING,13X,E12.5,5X,E12.5)
   19 FORMAT(2X,12HCONSTRUCTION,8X,E12.5,5X,E12.5)
   20 FORMAT(2X,12HPRETREATMENT,8X,E12.5,5X,E12.5,/)
   21 FORMAT(2X,5HTERMS,12X,11H1 ST EFFECT,6X,11H2 ND EFFECT,6X,11H3 RD
     1EFFECT,/)
   22 FORMAT(2X,10HTEMP. DROP,6X,E12.5,5X,E12.5,5X,E12.5)
   23 FORMAT(2X,11HWATER PROD.,5X,E12.5,5X,E12.5,5X,E12.5,/)
   24 FORMAT(2X,26HREAD NEW OPTIMIZATION DATA)
   25 FORMAT(2X,6HX1(4)=E12.5,11X,2HQ=E12.5,7X,6HX2(4)=E12.5)
   26 FORMAT(2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,/)
   27 FORMAT(2X,10HCOND. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   28 FORMAT(2X,10HPUMP. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   29 FORMAT(2X,8HB.P. EL.,8X,E12.5,5X,E12.5,5X,E12.5,/)
   31 FORMAT(2X,6HX1(4)=E12.5,2X,2HQ=E12.5,2X,3HTN=E12.5)
   33 FORMAT(2X,6HXF(I)=E12.5,2X,6HQF(I)=E12.5,2X,7HWCI(I)=E12.5,/)
   34 FORMAT(2X,3HTN=F4.1)
  123 FORMAT(2X,6HX1(4)=E12.5,2X,2HQ=E12.5,2X,3HTX=E12.5)
  124 FORMAT(3E12.5)
   71 READ 4,TX,TN,TA,TB,TR,TZ
  125 READ 124,X1(4),Q,ZZ
  126 READ 3,WC,TE(1),TE(2),TE(3),ER
      READ 3,Y1(2),Y1(3),Y1(4),Y3(2),Y3(3)
      READ 3,(DS(I),I=1,5)
      READ 4,(TV(I),I=1,5),ZZ
      READ 1,U,A,HT,HTS,X3(1),D,S(2),S(3),S(4),V
      READ 2,R,X1(1),CST,W,CHT,CCD,CPP
      READ 4,B,C5,PC,CP,CC,ERROR
      M=1
      L=1
   41 DO 42 I=2,3
      X=EXPF(-CP*Y3(I)/HT)
   42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
      Y3(4)=HT*LOGF((X1(3)+Y1(4)*X1(4))/(X1(4)*(1.+Y1(4))))/CP
      DO 43 I=2,4
      X3(I)=X3(I-1)+Y3(I)
   43 DT(I)=(Q/CP+TE(I-1)*(1.-X1(1)/X1(I-1)))/(1.+Y1(I)*X1(1)/X1(I-1))
```

Table A-3 Computer Program (2)

```
      FLASH DISTILLATION BY SIMPLEX METHOD AND MAXIMUM PRINCIPLE
      DIMENSION Y1(4),Y3(4),X1(4),X2(4),X3(4),DT(4),X1Y1(3),S(4)
      DIMENSION X1Y3(3),X2Y1(3),DTY1(3),XIY3(3),DTX1(2),X2X3(2),X2X1(2)
      DIMENSION WPR(4),DS(5),TV(5),PU,4),AR(4),TE(3),QF(6),XF(6),WCI(6)
      DIMENSION QF(6),XF(6),WCI(6)
    1 FORMAT(10F7.1)
    2 FORMAT(7E10.3)
    3 FORMAT(5E12.5)
    4 FORMAT(6E12.5)
    5 FORMAT(2X,37HTHE FOLLOWING ARE OPTIMUN OUTPUT DATA,/)
    6 FORMAT(2X,6HY1(2)=E12.5,7X,6HY1(3)=E12.5,7X,6HY1(4)=E12.5)
    7 FORMAT(2X,6HY3(2)=E12.5,7X,6HY3(3)=E12.5,7X,6HY3(4)=E12.5)
    8 FORMAT(2X,6HX1(1)=E12.5,7X,6HX1(2)=E12.5,7X,6HX1(3)=E12.5)
    9 FORMAT(2X,6HDT(2)=E12.5,7X,6HDT(3)=E12.5,7X,6HDT(4)=E12.5)
   10 FORMAT(2X,6HX2(2)=E12.5,7X,6HX2(3)=E12.5,7X,6HX2(4)=E12.5)
   11 FORMAT(2X,6HX3(1)=E12.5,7X,6HX3(2)=E12.5,7X,6HX3(3)=E12.5)
   12 FORMAT(2X,6HH1Y11=E12.5,7X,6HH2Y12=E12.5,7X,6HH3Y13=E12.5)
   13 FORMAT(2X,6HH1Y31=E12.5,7X,6HH2Y32=E12.5,7X,2HW=E12.5,/)
   14 FORMAT(2X,5HTERMS,20X,7HCOST($),7X,10HPERCENTAGE,/)
   15 FORMAT(2X,5HSTEAM,15X,E12.5,5X,E12.5)
   16 FORMAT(2X,6HHEATER,14X,E12.5,5X,E12.5)
   17 FORMAT(2X,15HCONDENSING AREA,5X,E12.5,5X,E12.5)
   18 FORMAT(2X,7HPUMPING,13X,E12.5,5X,E12.5)
   19 FORMAT(2X,12HCONSTRUCTION,8X,E12.5,5X,E12.5)
   20 FORMAT(2X,12HPRETREATMENT,8X,E12.5,5X,E12.5,/)
   21 FORMAT(2X,5HTERMS,12X,11H1 ST EFFECT,6X,11H2 ND EFFECT,6X,11H3 RD
     1EFFECT,/)
   22 FORMAT(2X,10HTEMP. DROP,6X,E12.5,5X,E12.5,5X,E12.5)
   23 FORMAT(2X,11HWATER PROD.,5X,E12.5,5X,E12.5,5X,E12.5,/)
   24 FORMAT(2X,26HREAD NEW OPTIMIZATION DATA)
   25 FORMAT(2X,6HX1(4)=E12.5,11X,2HQ=E12.5,7X,6HX2(4)=E12.5)
   26 FORMAT(2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,/)
   27 FORMAT(2X,10HCOND. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   28 FORMAT(2X,10HPUMP. COST,6X,E12.5,5X,E12.5,5X,E12.5)
   29 FORMAT(2X,8HB.P. EL.,8X,E12.5,5X,E12.5,5X,E12.5,/)
   31 FORMAT(2X,6HX1(4)=E12.5,2X,2HQ=E12.5,2X,3HTN=E12.5)
   33 FORMAT(2X,6HXF(I)=E12.5,2X,6HQF(I)=E12.5,2X,7HWCI(I)=E12.5,/)
   34 FORMAT(2X,3HTN=F4.1)
  123 FORMAT(2X,6HX1(4)=E12.5,2X,2HQ=E12.5,2X,3HTX=E12.5)
  124 FORMAT(3E12.5)
   71 READ 4,TX,TN,TA,TB,TR,TZ
  125 READ 124,X1(4),Q,ZZ
  126 READ 3,WC,TE(1),TE(2),TE(3),ER
      READ 3,Y1(2),Y1(3),Y1(4),Y3(2),Y3(3)
      READ 3,(DS(I),I=1,5)
      READ 4,(TV(I),I=1,5),ZZ
      READ 1,U,A,HT,HTS,X3(1),D,S(2),S(3),S(4),V
      READ 2,R,X1(1),CST,W,CHT,CCD,CPP
      READ 4,B,C5,PC,CP,CC,ERROR
      M=1
      L=1
   41 DO 42 I=2,3
      X=EXPF(-CP*Y3(I)/HT)
   42 X1(I)=(X*X1(I-1))/(1.+Y1(I)-Y1(I)*X)
      Y3(4)=HT*LOGF((X1(3)+Y1(4)*X1(4))/(X1(4)*(1.+Y1(4))))/CP
      DO 43 I=2,4
      X3(I)=X3(I-1)+Y3(I)
   43 DT(I)=(Q/CP+TE(I-1)*(1.-X1(1)/X1(I-1)))/(1.+Y1(I)*X1(1)/X1(I-1))
```

```
      WA=W/(1.-X1(1)/X1(4))
      C2=CHT*Q*WA/(U*(A-X4(1)+0.5*DT(2)))
      C1=CST*Q*WA/HTS
      X3(1)=C2+C5+C1
      DO 44 I=2,4
      WPR(I-1)=WA*X1(1)*(1./X1(I-1)-1./X1(I))
      X=WPR(I-1)*HT/U
      AR(I)=CCD*X/(DT(I)-TE(I-1)+Y3(I)/(2.*S(I)))
      Y=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      PU(I)=CPP*X1(1)*Y1(I)*B*Y*WA/(D*X1(I-1))
      X2(I)=X2(I-1)+AR(I)+PU(I)
      X1Y1(I-1)=X1(I)*(X1(I)-X1(I-1))/(X1(I-1)*(1.+Y1(I)))
      X1Y3(I-1)=-CP*X1(I)*(X1(I-1)+Y1(I)*X1(I))/(HT*X1(I-1))
      DTY1(I-1)=-DT(I)/(X1(I-1)/X1(1)+Y1(I))
      XB1=X1(1)*X1Y1(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
      XB3=X1(1)*X2Y1(I-1)*(1./X1(I-1)-1./X1(I))
      XB2=XB3/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      DT(1)=CCD*HT*WA*(XB1-XB2)/U
      YA1=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      Y1(1)=CPP*X1(1)*B*YA1*WA/(D*X1(I-1))
      X2Y1(I-1)=DT(1)+Y1(1)
      XB1=X1(1)*X1Y3(I-1)/((X1(I)**2)*(X2(I)-TE(I-1)+0.5*Y3(I)/S(I)))
      XB3=X1(1)*(1./X1(I-1)-1./X1(I))
      XB2=XB3/(2.0*S(I)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      XA=HT*EXPF(-HT/(R*(X3(I-1)+Y3(I))))/(R*(X3(I-1)+Y3(I))**2)
   44 X2Y3(I-1)=CCD*HT*WA*(XB1-XB2)/U+Y
      BT=X2(3)+Y3(4)-545.
      CT=(Q*WA+TE(3)*W)/BT
      C6=(PC-CC)*WA+CC*CT
      X2(4)=X2(4)+C6
      AR(1)=AR(2)+AR(3)+AR(4)
      PU(1)=PU(2)+PU(3)+PU(4)
      WPR(4)=WPR(1)+WPR(2)+WPR(3)
      PC1=C1*100./X2(4)
      PC2=C2*100./X2(4)
      PC3=AR(1)*100./X2(4)
      PC4=PU(1)*100./X2(4)
      PC5=C5*100./X2(4)
      PC6=C6*100./X2(4)
      TD1=X3(1)-X3(2)
      TD2=X3(2)-X3(3)
      TD3=X3(3)-X3(4)
      XB1=-CCD*W*X1(1)*X1Y1(3)*V*HT*X1(1)*(1./X1(3)-1./X1(4))/U
      XB2=XB1/(((X1(4)-X1(1))**2)*(DT(4)-TE(3)+0.5*Y3(4)/S(4)))
      X5A=-CPP*X1(1)*Y1(4)*B*X1(1)*X1Y1(3)*W/((X1(4)-X1(1))**2)
      X2Y1(3)=X2Y1(3)+XB2+X5A*YA1/(D*X1(3))
      PT=-WA*X1(1)/(X1(4)*(X1(4)-X1(1)))
      AT=PT*X1Y1(3)
      X2Y1(3)=X2Y1(3)+(PC-CC)*AT+CC*Q*AT/BT
      X2Y1(1)=X2Y1(1)-CHT*Q*WA*DTY1(1)/(U*TE(1)*(A-X3(1)+0.5*DT(2))**2)
      XB1=X1(1)*(1.0/X1(3)-1.0/X1(4))*HT/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=-(W*X1(1)*X1Y3(3)*V/((X1(4)-X1(1))**2))*XB1*CCD/U
      X=W*X1(1)*X1Y3(3)*YA1*B*X1(1)*Y1(4)/((X1(4)-X1(1))**2)
      X2Y3(3)=X3Y2(3)+XB2-CPP*X/(D*X1(3))
      X2Y3(3)=X2Y3(3)+(PC-CC)*PT*X1Y3(3)+CC*(Q*PT*X1Y3(3)/BT-CT/BT)
      DO 45 I=3,4
      K=I-2
      XQ=1.0*X1(1)*(TE(I-1)+DT(I)*Y1(I))
```

```
      WA=W/(1.-X1(1)/X1(4))
      C2=CHT*Q*WA/(U*(A-X4(1)+0.5*DT(2)))
      C1=CST*Q*WA/HTS
      X3(1)=C2+C5+C1
      DO 44 I=2,4
      WPR(I-1)=WA*X1(1)*(1./X1(I-1)-1./X1(I))
      X=WPR(I-1)*HT/U
      AR(I)=CCD*X/(DT(I)-TE(I-1)+Y3(I)/(2.*S(I)))
      Y=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      PU(I)=CPP*X1(1)*Y1(I)*B*Y*WA/(D*X1(I-1))
      X2(I)=X2(I-1)+AR(I)+PU(I)
      X1Y1(I-1)=X1(I)*(X1(I)-X1(I-1))/(X1(I-1)*(1.+Y1(I)))
      X1Y3(I-1)=-CP*X1(I)*(X1(I-1)+Y1(I)*X1(I))/(HT*X1(I-1))
      DTY1(I-1)=-DT(I)/(X1(I-1)/X1(1)+Y1(I))
      XB1=X1(1)*X1Y1(I-1)/((X1(I)**2)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I)))
      XB3=X1(1)*X2Y1(I-1)*(1./X1(I-1)-1./X1(I))
      XB2=XB3/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      DT(1)=CCD*HT*WA*(XB1-XB2)/U
      YA1=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      Y1(1)=CPP*X1(1)*B*YA1*WA/(D*X1(I-1))
      X2Y1(I-1)=DT(1)+Y1(1)
      XB1=X1(1)*X1Y3(I-1)/((X1(I)**2)*(X2(I)-TE(I-1)+0.5*Y3(I)/S(I)))
      XB3=X1(1)*(1./X1(I-1)-1./X1(I))
      XB2=XB3/(2.0*S(I)*(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      XA=HT*EXPF(-HT/(R*(X3(I-1)+Y3(I))))/(R*(X3(I-1)+Y3(I))**2)
   44 X2Y3(I-1)=CCD*HT*WA*(XB1-XB2)/U+Y
      BT=X2(3)+Y3(4)-545.
      CT=(Q*WA+TE(3)*W)/BT
      C6=(PC-CC)*WA+CC*CT
      X2(4)=X2(4)+C6
      AR(1)=AR(2)+AR(3)+AR(4)
      PU(1)=PU(2)+PU(3)+PU(4)
      WPR(4)=WPR(1)+WPR(2)+WPR(3)
      PC1=C1*100./X2(4)
      PC2=C2*100./X2(4)
      PC3=AR(1)*100./X2(4)
      PC4=PU(1)*100./X2(4)
      PC5=C5*100./X2(4)
      PC6=C6*100./X2(4)
      TD1=X3(1)-X3(2)
      TD2=X3(2)-X3(3)
      TD3=X3(3)-X3(4)
      XB1=-CCD*W*X1(1)*X1Y1(3)*V*HT*X1(1)*(1./X1(3)-1./X1(4))/U
      XB2=XB1/(((X1(4)-X1(1))**2)*(DT(4)-TE(3)+0.5*Y3(4)/S(4)))
      X5A=-CPP*X1(1)*Y1(4)*B*X1(I)*X1Y1(3)*W/((X1(4)-X1(1))**2)
      X2Y1(3)=X2Y1(3)+XB2+X5A*YA1/(D*X1(3))
      PT=-WA*X1(1)/(X1(4)*(X1(4)-X1(1)))
      AT=PT*X1Y1(3)
      X2Y1(3)=X2Y1(3)+(PC-CC)*AT+CC*Q*AT/BT
      X2Y1(1)=X2Y1(1)-CHT*Q*WA*DTY1(1)/(U*TE(1)*(A-X3(1)+0.5*DT(2))**2)
      XB1=X1(1)*(1.0/X1(3)-1.0/X1(4))*HT/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=-(W*X1(1)*X1Y3(3)*V/((X1(4)-X1(1))**2))*XB1*CCD/U
      X=W*X1(1)*X1Y3(3)*YA1*B*X1(1)*Y1(4)/((X1(4)-X1(1))**2)
      X2Y3(3)=X3Y2(3)+XB2-CPP*X/(D*X1(3))
      X2Y3(3)=X2Y3(3)+(PC-CC)*PT*X1Y3(3)+CC*(Q*PT*X1Y3(3)/BT-CT/BT)
      DO 45 I=3,4
      K=I-2
      XQ=1.0*X1(1)*(TE(I-1)+DT(I)*Y1(I))
```

```
      DTX1(K)=XQ/(X1(I-1)*(X1(I-1)+Y1(I)*X1(1)))
      X=(-1./(X1(I-1)**2)+1./(X1(I-1)*X1(I)))
      X=X/(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))
      YA=DTX1(K)*(1./X1(I-1)-1./X1(I))
      YA=YA/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      XA=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      Y=-CPP*X1(1)*Y1(I)*B*XA*WA/(D*X1(I-1)**2)
      X2X1(K)=Y+CCD*HT*WA*X1(1)*(X-YA)/U
      X5=EXPF(-HT/(R*X3(I-1)))/(X3(I-1)**2)
      YA=EXPF(-HT/(R*(X3(I-1)+Y3(I))))/((X3(I-1)+Y3(I))**2)
   45 X2X3(K)=1.0*CPP*X1(1)*Y1(I)*WA*B*HT*(X5-YA)/(R*D*X1(I-1))
      XB1=X1(1)*HT*(1.0/X1(3)-1.0/X1(4))/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=W*X1(1)*X1(4)/(X1(3)*(X1(4)-X1(1))**2)
      Y=-CPP*X1(1)*Y1(4)*B*XB2*XA/(D*X1(3))
      X2X1(2)=X2X1(2)+Y-CCD*XB2*XB1/U
      X2X1(2)=X2X1(2)+(PC-CC)*PT*X1(4)/X1(3)+CC*Q*PT*X1(4)/(X1(3)*BT)
      X2X3(2)=X2X3(2)-CC*CT/BT
      Z13=-X2Y3(3)/X1Y3(3)
      Z12=Z13*X1(4)/X1(3)+X2X1(2)
      Z32=X2X3(2)
      Z11=Z12*X1(3)/X1(2)+X2X1(1)
      Z31=X2X3(2)
      H1Y11=Z11*X1Y1(1)+X2Y1(1)
      H1Y31=Z11*X1Y3(1)+X2Y3(1)+Z31
      H2Y12=Z12*X1Y1(2)+X2Y1(2)
      H2Y32=Z12*X1Y3(2)+X2Y3(2)+Z32
      H3Y13=Z13*X1Y1(3)+X2Y1(3)
      IF(M-1)56,66,67
   66 TE(1)=1.01+((X1(1)+Y1(2)*X1(2))/(1.+Y1(2))+X1(2))/0.06
      TE(2)=1.0075+((X1(2)+Y1(3)*X1(3))/(1.+Y1(3))+X1(3))/0.0694
      TE(3)=0.32+((X1(3)+Y1(4)*X1(4))/(1.+Y1(4))+X1(4))/0.0630
      M=M+1
      GO TO 41
   67 M=1
      IF(ABSF(H1Y11)-ER)47,47,51
   47 IF(ABSF(H1Y31)-ER)48,48,51
   48 IF(ABSF(H2Y12)-ER)49,49,51
   49 IF(ABSF(H2Y32)-ER)50,50,51
   50 IF(ABSF(H3Y13)-ER)68,68,51
   51 IF (H1Y11) 137,137,138
  137 TV(1)=-1.
  138 IF (H2Y12) 139,139,140
  139 TV(2)=-1.
  140 IF (H3Y13) 141,141,142
  141 TV(3)=-1.
  142 IF (H1Y31) 143,143,144
  143 TV(4)=-1.
  144 IF (H2Y32) 145,145,146
  145 TV(5)=-1.
  146 IF(WC-X3(4))36,36,35
   36 DO 37 I=1,5
   37 DS(I)=0.5*DS(I)
      Y1(2)=Y11T
      Y1(3)=Y12T
      Y1(4)=Y13T
      Y3(2)=Y31T
      Y3(3)=Y32T
      GO TO 38
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      DTX1(K)=XQ/(X1(I-1)*(X1(I-1)+Y1(I)*X1(1)))
      X=(-1./(X1(I-1)**2)+1./(X1(I-1)*X1(I)))
      X=X/(DT(I)-TE(I-1)+0.5*Y3(I)/S(I))
      YA=DTX1(K)*(1./X1(I-1)-1./X1(I))
      YA=YA/((DT(I)-TE(I-1)+0.5*Y3(I)/S(I))**2)
      XA=EXPF(-HT/(R*X3(I-1)))-EXPF(-HT/(R*(X3(I-1)+Y3(I))))
      Y=-CPP*X1(1)*Y1(I)*B*XA*WA/(D*X1(I-1)**2)
      X2X1(K)=Y+CCD*HT*WA*X1(1)*(X-YA)/U
      X5=EXPF(-HT/(R*X3(I-1)))/(X3(I-1)**2)
      YA=EXPF(-HT/(R*(X3(I-1)+Y3(I))))/((X3(I-1)+Y3(I))**2)
   45 X2X3(K)=1.0*CPP*X1(1)*Y1(I)*WA*B*HT*(X5-YA)/(R*D*X1(I-1))
      XB1=X1(1)*HT*(1.0/X1(3)-1.0/X1(4))/(DT(4)-TE(3)+0.5*Y3(4)/S(4))
      XB2=W*X1(1)*X1(4)/(X1(3)*(X1(4)-X1(1))**2)
      Y=-CPP*X1(1)*Y1(4)*B*XB2*XA/(D*X1(3))
      X2X1(2)=X2X1(2)+Y-CCD*XB2*XB1/U
      X2X1(2)=X2X1(2)+(PC-CC)*PT*X1(4)/X1(3)+CC*Q*PT*X1(4)/(X1(3)*BT)
      X2X3(2)=X2X3(2)-CC*CT/BT
      Z13=-X2Y3(3)/X1Y3(3)
      Z12=Z13*X1(4)/X1(3)+X2X1(2)
      Z32=X2X3(2)
      Z11=Z12*X1(3)/X1(2)+X2X1(1)
      Z31=X2X3(2)
      H1Y11=Z11*X1Y1(1)+X2Y1(1)
      H1Y31=Z11*X1Y3(1)+X2Y3(1)+Z31
      H2Y12=Z12*X1Y1(2)+X2Y1(2)
      H2Y32=Z12*X1Y3(2)+X2Y3(2)+Z32
      H3Y13=Z13*X1Y1(3)+X2Y1(3)
      IF(M-1)66,66,67
   66 TE(1)=1.01+((X1(1)+Y1(2)*X1(2))/(1.+Y1(2))+X1(2))/0.06
      TE(2)=1.0075+((X1(2)+Y1(3)*X1(3))/(1.+Y1(3))+X1(3))/0.0694
      TE(3)=0.32+((X1(3)+Y1(4)*X1(4))/(1.+Y1(4))+X1(4))/0.0630
      M=M+1
      GO TO 41
   67 M=1
      IF(ABSF(H1Y11)-ER)47,47,51
   47 IF(ABSF(H1Y31)-ER)48,48,51
   48 IF(ABSF(H2Y12)-ER)49,49,51
   49 IF(ABSF(H2Y32)-ER)50,50,51
   50 IF(ABSF(H3Y13)-ER)68,68,51
   51 IF (H1Y11) 137,137,138
  137 TV(1)=-1.
  138 IF (H2Y12) 139,139,140
  139 TV(2)=-1.
  140 IF (H3Y13) 141,141,142
  141 TV(3)=-1.
  142 IF (H1Y31) 143,143,144
  143 TV(4)=-1.
  144 IF (H2Y32) 145,145,146
  145 TV(5)=-1.
  146 IF(WC-X3(4))36,36,35
   36 DO 37 I=1,5
   37 DS(I)=0.5*DS(I)
      Y1(2)=Y11T
      Y1(3)=Y12T
      Y1(4)=Y13T
      Y3(2)=Y31T
      Y3(3)=Y32T
      GO TO 38
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 35 WC=X3(4)
 38 Y11T=Y1(2)
    Y12T=Y1(3)
    Y13T=Y1(4)
    Y31T=Y3(2)
    Y32T=Y3(3)
    Y1(2)=Y1(2)-TV(1)*DS(1)
    Y1(3)=Y1(3)-TV(2)*DS(2)
    Y1(4)=Y1(4)-TV(3)*DS(3)
    Y3(2)=Y3(2)-TV(4)*DS(4)
    Y3(3)=Y3(3)-TV(5)*DS(5)
    DO 32 I=1,5
 32 TV(I)=1.
    GO TO 41
 68 IF(TX-1.)69,69,63
 69 IF(TN-1.)72,78,79
 72 IF(TZ-1.)150,151,152
150 QF(1)=Q
    XF(1)=X1(4)
    WCI(1)=X3(4)
    TZ=1.
    GO TO 125
151 QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
    TZ=2.
    GO TO 125
152 QF(3)=Q
    XF(3)=X1(4)
    WCI(3)=X3(4)
 98 IF(WCI(1)-WCI(2))73,73,74
 74 Q=QF(1)
    X1(4)=XF(1)
    X3(4)=WCI(1)
    QF(1)=QF(2)
    XF(1)=XF(2)
    WCI(1)=WCI(2)
    QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
 73 IF(WCI(2)-WCI(3))75,75,76
 76 Q=QF(3)
    X1(4)=XF(3)
    X3(4)=WCI(3)
    QF(3)=QF(2)
    XF(3)=XF(2)
    WCI(3)=WCI(2)
    QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
    IF(WCI(2)-WCI(1))77,77,75
 77 Q=QF(2)
    X1(4)=XF(2)
    X3(4)=WCI(2)
    QF(2)=QF(1)
    XF(2)=XF(1)
    WCI(2)=WCI(1)
    QF(1)=Q
```

```
 35 WC=X3(4)
 38 Y11T=Y1(2)
    Y12T=Y1(3)
    Y13T=Y1(4)
    Y31T=Y3(2)
    Y32T=Y3(3)
    Y1(2)=Y1(2)-TV(1)*DS(1)
    Y1(3)=Y1(3)-TV(2)*DS(2)
    Y1(4)=Y1(4)-TV(3)*DS(3)
    Y3(2)=Y3(2)-TV(4)*DS(4)
    Y3(3)=Y3(3)-TV(5)*DS(5)
    DO 32 I=1,5
 32 TV(I)=1.
    GO TO 41
 68 IF(TX-1.)69,69,63
 69 IF(TN-1.)72,78,79
 72 IF(TZ-1.)150,151,152
150 QF(1)=Q
    XF(1)=X1(4)
    WCI(1)=X3(4)
    TZ=1.
    GO TO 125
151 QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
    TZ=2.
    GO TO 125
152 QF(3)=Q
    XF(3)=X1(4)
    WCI(3)=X3(4)
 98 IF(WCI(1)-WCI(2))73,73,74
 74 Q=QF(1)
    X1(4)=XF(1)
    X3(4)=WCI(1)
    QF(1)=QF(2)
    XF(1)=XF(2)
    WCI(1)=WCI(2)
    QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
 73 IF(WCI(2)-WCI(3))75,75,76
 76 Q=QF(3)
    X1(4)=XF(3)
    X3(4)=WCI(3)
    QF(3)=QF(2)
    XF(3)=XF(2)
    WCI(3)=WCI(2)
    QF(2)=Q
    XF(2)=X1(4)
    WCI(2)=X3(4)
    IF(WCI(2)-WCI(1))77,77,75
 77 Q=QF(2)
    X1(4)=XF(2)
    X3(4)=WCI(2)
    QF(2)=QF(1)
    XF(2)=XF(1)
    WCI(2)=WCI(1)
    QF(1)=Q
```

Table A-3 (continued)

```
      XF(1)=X1(4)
      WCI(1)=X3(4)
   75 IF(TX-1.)130,130,131
  130 TN=1.
      QF(4)=(QF(1)+QF(2))*0.5
      XF(4)=(XF(1)+XF(2))*0.5
      Q=QF(4)
      X1(4)=XF(4)
      PRINT 31,X1(4),Q,TN
      GO TO 126
   78 WCI(4)=X3(4)
      QF(5)=QF(4)+TA*(QF(4)-QF(3))
      XF(5)=XF(4)+TA*(XF(4)-XF(3))
      Q=QF(5)
      X1(4)=XF(5)
      TN=2.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   79 IF(TN-3.)80,83,90
   80 WCI(5)=X3(4)
      IF(WCI(5)-WCI(1))81,82,82
   81 QF(6)=QF(4)+TR*(QF(5)-QF(4))
      XF(6)=QF(4)+TR*(XF(5)-QF(4))
      Q=QF(6)
      X1(4)=XF(6)
      TN=3.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   83 WCI(6)=X3(4)
      IF(WCI(6)-WCI(1))84,86,86
   84 QF(3)=QF(6)
      XF(3)=XF(6)
      WCI(3)=WCI(6)
      GO TO 96
   82 IF(WCI(5)-WCI(2))86,86,87
   86 QF(3)=QF(5)
      XF(3)=XF(5)
      WCI(3)=WCI(5)
      GO TO 96
   87 IF(WCI(5)-WCI(3))88,88,89
   88 QF(3)=QF(5)
      XF(3)=XF(5)
      WCI(3)=WCI(5)
   89 QF(6)=QF(4)+TB*(QF(3)-QF(4))
      XF(6)=XF(4)+TB*(XF(3)-XF(4))
      Q=QF(6)
      X1(4)=XF(6)
      TN=4.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   90 IF(TN-5.)91,94,95
   91 WCI(6)=X3(4)
      IF(WCI(6)-WCI(3))92,92,93
   92 QF(3)=QF(6)
      XF(3)=XF(6)
      WCI(3)=WCI(6)
      GO TO 96
   93 QF(3)=0.5*(QF(3)+QF(1))
```

Table A-3 (continued)

```
      XF(1)=X1(4)
      WCI(1)=X3(4)
   75 IF(TX-1.)130,130,131
  130 TN=1.
      QF(4)=(QF(1)+QF(2))*0.5
      XF(4)=(XF(1)+XF(2))*0.5
      Q=QF(4)
      X1(4)=XF(4)
      PRINT 31,X1(4),Q,TN
      GO TO 126
   78 WCI(4)=X3(4)
      QF(5)=QF(4)+TA*(QF(4)-QF(3))
      XF(5)=XF(4)+TA*(XF(4)-XF(3))
      Q=QF(5)
      X1(4)=XF(5)
      TN=2.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   79 IF(TN-3.)80,83,90
   80 WCI(5)=X3(4)
      IF(WCI(5)-WCI(1))81,82,82
   81 QF(6)=QF(4)+TR*(QF(5)-QF(4))
      XF(6)=QF(4)+TR*(XF(5)-QF(4))
      Q=QF(6)
      X1(4)=XF(6)
      TN=3.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   83 WCI(6)=X3(4)
      IF(WCI(6)-WCI(1))84,86,86
   84 QF(3)=QF(6)
      XF(3)=XF(6)
      WCI(3)=WCI(6)
      GO TO 96
   82 IF(WCI(5)-WCI(2))86,86,87
   86 QF(3)=QF(5)
      XF(3)=XF(5)
      WCI(3)=WCI(5)
      GO TO 96
   87 IF(WCI(5)-WCI(3))88,88,89
   88 QF(3)=QF(5)
      XF(3)=XF(5)
      WCI(3)=WCI(5)
   89 QF(6)=QF(4)+TB*(QF(3)-QF(4))
      XF(6)=XF(4)+TB*(XF(3)-XF(4))
      Q=QF(6)
      X1(4)=XF(6)
      TN=4.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   90 IF(TN-5.)91,94,95
   91 WCI(6)=X3(4)
      IF(WCI(6)-WCI(3))92,92,93
   92 QF(3)=QF(6)
      XF(3)=XF(6)
      WCI(3)=WCI(6)
      GO TO 96
   93 QF(3)=0.5*(QF(3)+QF(1))
```

```
      XF(3)=0.5*(XF(3)+XF(1))
      Q=QF(3)
      X1(4)=XF(3)
      TN=5.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   94 WCI(3)=X3(4)
      QF(2)=0.5*(QF(2)+QF(1))
      XF(2)=0.5*(XF(2)+XF(1))
      Q=QF(2)
      X1(4)=XF(2)
      TN=6.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   95 WCI(2)=X3(4)
   96 TEST=(((WCI(1)-WCI(4))**2.+(WCI(2)-WCI(4))**2.+(WCI(3)-WCI(4))**2.
     1)/3.)**.5
      PUNCH 33,(XF(I),QF(I),WCI(I),I=1,3)
      IF(TEST-ERROR)97,97,98
   97 TX=2.
      GO TO 98
  131 Q=QF(1)
      X1(4)=XF(1)
      PRINT 31,X1(4),Q,TX
      GO TO 126
   63 PUNCH 5
      PUNCH 25,X1(4),Q,X2(4)
      PUNCH 6,Y1(2),Y1(3),Y1(4)
      PUNCH 7,Y3(2),Y3(3),Y3(4)
      PUNCH 8,X1(1),X1(2),X1(3)
      PUNCH 9,DT(2),DT(3),DT(4)
      DO 64 I=2,4
   64 X3(I)=X3(I)-460.
      PUNCH 10,X2(1),X2(2),X2(3)
      PUNCH 11,X3(1),X3(2),X3(3)
      PUNCH 12,H1Y11,H2Y12,H3Y13
      PUNCH 13,H1Y31,H2Y32,WPR(4)
      PUNCH 14
      PUNCH 15,C1,PC1
      PUNCH 16,C2,PC2
      PUNCH 17,AR(1),PC3
      PUNCH 18,PU(1),PC4
      PUNCH 19,C5,PC5
      PUNCH 20,C6,PC6
      PUNCH 21
      PUNCH 22,TD1,TD2,TD3
      PUNCH 23,WPR(1),WPR(2),WPR(3)
      PUNCH 27,AR(2),AR(3),AR(4)
      PUNCH 28,PU(2),PU(3),PU(4)
      PUNCH 29,TE(1),TE(2),TE(3)
      PUNCH 24
      GO TO 71
      END
```

Table A-3 (continued)

```
      XF(3)=0.5*(XF(3)+XF(1))
      Q=QF(3)
      X1(4)=XF(3)
      TN=5.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   94 WCI(3)=X3(4)
      QF(2)=0.5*(QF(2)+QF(1))
      XF(2)=0.5*(XF(2)+XF(1))
      Q=QF(2)
      X1(4)=XF(2)
      TN=6.
      PRINT 31,X1(4),Q,TN
      GO TO 126
   95 WCI(2)=X3(4)
   96 TEST=(((WCI(1)-WCI(4))**2.+(WCI(2)-WCI(4))**2.+(WCI(3)-WCI(4))**2.
     1)/3.)**.5
      PUNCH 33,(XF(I),QF(I),WCI(I),I=1,3)
      IF(TEST-ERROR)97,97,98
   97 TX=2.
      GO TO 98
  131 Q=QF(1)
      X1(4)=XF(1)
      PRINT 31,X1(4),Q,TX
      GO TO 126
   63 PUNCH 5
      PUNCH 25,X1(4),Q,X2(4)
      PUNCH 6,Y1(2),Y1(3),Y1(4)
      PUNCH 7,Y3(2),Y3(3),Y3(4)
      PUNCH 8,X1(1),X1(2),X1(3)
      PUNCH 9,DT(2),DT(3),DT(4)
      DO 64 I=2,4
   64 X3(I)=X3(I)-460.
      PUNCH 10,X2(1),X2(2),X2(3)
      PUNCH 11,X3(1),X3(2),X3(3)
      PUNCH 12,H1Y11,H2Y12,H3Y13
      PUNCH 13,H1Y31,H2Y32,WPR(4)
      PUNCH 14
      PUNCH 15,C1,PC1
      PUNCH 16,C2,PC2
      PUNCH 17,AR(1),PC3
      PUNCH 18,PU(1),PC4
      PUNCH 19,C5,PC5
      PUNCH 20,C6,PC6
      PUNCH 21
      PUNCH 22,TD1,TD2,TD3
      PUNCH 23,WPR(1),WPR(2),WPR(3)
      PUNCH 27,AR(2),AR(3),AR(4)
      PUNCH 28,PU(2),PU(3),PU(4)
      PUNCH 29,TE(1),TE(2),TE(3)
      PUNCH 24
      GO TO 71
      END
```

Table A-4 Input Data and Sample Output Results

```
THE FOLLOWING ARE INPUT DATA

 X1(4)            Q              WC           TE(1)          TE(2)          TE(3)
.06.00E-00.2.  000E+01 1.00000E-00 2.30350E-00 2.39360E-00 2.26160E-00
 Y1(2)          Y1(3)          Y1(4)          Y3(2)          Y3(3)
.24029E+01 0.32712E+01 0.44967E+01-0.54337E+02-0.48097E+02
 DS(1)          DS(2)          DS(3)          DS(4)          DS(5)          ER
5.00000E-01 5.00000E-01 5.00000E-01 1.00000E-00 1.00000E-00 1.00000E-04
  U       A       HT      HTS     X4(1)     D      S(2)    S(3)    S(4)     V
 510.0  734.4 1000.0  928.9  710.0   62.5   23.0   23.0   22.0    1.0
   R         X1(1)          CST            W             CHT           CCD           CPP
1.104E-01 0.035E+00 0.025E-02 8.340E+05 0.376E-04 2.397E-05 2.903E-09
   B             C5             PC             CP             CC
1.79000E+09 2.08000E-02 1.79600E-06 1.00000E-00 5.98750E-07


THE FOLLOWING ARE OPTIMUM OUTPUT DATA

X1(4)= 6.50000E-02           Q= 2.70000E+01          X3(4)= 2.85577E-01
Y1(2)= 2.13689E-00       Y1(3)= 2.67673E-00          Y1(4)= 3.88091E-00
Y3(2)=-5.40402E+01       Y3(3)=-4.80402E+01          Y3(4)=-4.45277E+01
X1(1)= 3.50000E-02       X1(2)= 4.19224E-02          X1(3)= 5.12378E-02
DT(2)= 8.60176E-00       DT(3)= 8.05599E-00          DT(4)= 7.62869E-00
X2(1)= 1.53361E-01       X2(2)= 1.87120E-01          X2(3)= 2.17784E-01
X3(2)= 1.95959E+02       X3(3)= 1.47919E+02          X3(4)= 1.03391E-02
H1Y11= 7.27000E-07       H2Y12= 2.80440E-05          H3Y13= 1.53270E-05
H1Y31= 6.39843E-05       H2Y32= 1.70280E-05


ITEM                      COST($)         PERCENTAGE

STEAM                   1.31308E-01     4.59799E+01
HEATER                  1.25326E-03     4.38853E-01
CONDENSING AREA         8.47040E-02     2.96605E+01
PUMPING                 9.35936E-03     3.27734E-00
CONSTRUCTION            2.08000E-02     7.28348E-00
PRETREATMENT            3.81525E-02     1.33597E+01

TERMS               1 ST EFFECT       2 ND EFFECT       3 RD EFFECT

TEMP. DROP          5.40402E+01       4.80402E+01       4.45277E+01
WATER PROD.         2.96360E+03       2.74322E+03       2.61297E+03

COND. COST          2.77396E-02       2.82511E-02       2.87133E-02
PUMP. COST          6.01949E-03       2.41289E-03       9.26982E-04
B.P. EL.            2.37144E-00       2.44787E-00       2.33779E-00
```

Table A-4 Input Data and Sample Output Results

```
THE FOLLOWING ARE INPUT DATA

 X1(4)              Q               WC            TE(1)          TE(2)          TE(3)
.06.00E-00.2.  000E+01 1.00000E-00 2.30350E-00 2.39360E-00 2.26160E-00
 Y1(2)          Y1(3)          Y1(4)          Y3(2)          Y3(3)
.24029E+01 0.32712E+01 0.44967E+01-0.54337E+02-0.48097E+02
 DS(1)          DS(2)          DS(3)          DS(4)          DS(5)          ER
5.00000E-01 5.00000E-01 5.00000E-01 1.00000E-00 1.00000E-00 1.00000E-04
  U       A      HT     HTS    X4(1)     D      S(2)    S(3)    S(4)     V
 510.0  734.4 1000.0  928.9  710.0   62.5   23.0    23.0    22.0    1.0
   R        X1(1)       CST          W          CHT         CCD         CPP
1.104E-01 0.035E+00 0.025E-02 8.340E+03 0.376E-04 2.397E-03 2.903E-09
   B          C5          PC          CP          CC
1.79000E+09 2.08000E-02 1.79600E-06 1.00000E-00 5.98750E-07

THE FOLLOWING ARE OPTIMUM OUTPUT DATA

X1(4)= 6.50000E-02          Q= 2.70000E+01        X3(4)= 2.85577E-01
Y1(2)= 2.13689E-00      Y1(3)= 2.67673E-00        Y1(4)= 3.88091E-00
Y3(2)=-5.40402E+01      Y3(3)=-4.80402E+01        Y3(4)=-4.45277E+01
X1(1)= 3.50000E-02      X1(2)= 4.19224E-02        X1(3)= 5.12378E-02
DT(2)= 6.60176E-00      DT(3)= 8.05599E-00        DT(4)= 7.62669E-00
X2(1)= 1.53361E-01      X2(2)= 1.87120E-01        X2(3)= 2.17784E-01
X3(2)= 1.95959E+02      X3(3)= 1.47919E+02        X3(4)= 1.03391E-02
H1Y11= 7.27000E-07      H2Y12= 2.80440E-05        H3Y13= 1.53270E-05
H1Y31= 6.39643E-05      H2Y32= 1.70280E-05

ITEM                      COST($)         PERCENTAGE

STEAM                  1.31308E-01      4.59799E+01
HEATER                 1.25326E-03      4.38853E-01
CONDENSING AREA        8.47040E-02      2.96605E+01
PUMPING                9.35936E-03      3.27734E-00
CONSTRUCTION           2.08000E-02      7.28348E-00
PRETREATMENT           3.81525E-02      1.33597E+01

ITEMS              1 ST EFFECT      2 ND EFFECT      3 RD EFFECT

TEMP. DROP         5.40402E+01      4.80402E+01      4.45277E+01
WATER PROD.        2.96360E+03      2.74322E+03      2.61297E+03

COND. COST         2.77396E-02      2.82511E-02      2.87133E-02
PUMP. COST         6.01949E-03      2.41269E-03      9.26962E-04
B.P. EL.           2.37144E-00      2.44787E-00      2.33779E-00
```

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by

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In PART ONE a detailed analysis of a MEMS process is made and a mathematical model of the process is developed. An optimization study of such a model is carried out by a discrete analog of the maximum principle in conjunction with two search techniques: the parametric search and the simplex method. Both methods lead to the same optimal results. In contrast to the parametric search, the simplex method gives rise directly to the optimum point. The parametric search, however, gives detailed information about the influences of the individual parameters on the water cost and the other operating variables. In PART TWO a general mathematical model of a sequential multistage reverse osmosis process is developed. This model is obtained under the assumption of plug flow inside the tubular osmosis unit to take into account the brine concentration changes along the membrane tube. Several simplified versions of this model are also proposed.

In PART ONE a detailed analysis of a MEMS process is made and a mathematical model of the process is developed. An optimization study of such a model is carried out by a discrete analog of the maximum principle in conjunction with two search techniques: the parametric search and the simplex method. Both methods lead to the same optimal results. In contrast to the parametric search, the simplex method gives rise directly to the optimum point. The parametric search, however, gives detailed information about the influences of the individual parameters on the water cost and the other operating variables. In PART TWO a general mathematical model of a sequential multistage reverse osmosis process is developed. This model is obtained under the assumption of plug flow inside the tubular osmosis unit to take into account the brine concentration changes along the membrane tube. Several simplified versions of this model are also proposed.